X. *Experimental Researches in Magnetism.*

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[PLATES 57–68.]

§ 1. THE experiments to be described in this paper deal with a number of parts of one general subject—the magnetisation of iron and steel. They relate to—

(1.) The magnetic susceptibility of iron and steel; the form of the curve of magnetisation and magnetising force; and the changes of magnetism caused by gradual reversal and other cyclic changes of magnetising force.
(2.) The influence of vibration on magnetic susceptibility and retentiveness.
(3.) The influence of permanent strain on magnetic susceptibility and retentiveness.
(4.) The energy expended in producing cyclic changes of magnetisation.
(5.) The ratio of residual to induced magnetism.
(6.) The changes of induced and residual magnetism caused by changes of stress.
(7.) The effects of constant stress on magnetic susceptibility and retentiveness.
(8.) The changes of magnetism caused by changes of temperature.
(9.) The effects of temperature on magnetic susceptibility.

In connexion with the above, experiments of a closely related character have been made on the following additional subjects, but an account of them will be reserved for another paper:—

(10.) The changes of thermoelectric quality caused by changes of stress.
(11.) The changes of thermoelectric quality caused by magnetisation.*

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* The following previous publications may be referred to as preliminary notices of some of the results:—

(1.) "Effects of Stress on the Thermoelectric Quality of Metals" (Abstract), Proc. Roy. Soc., No. 214, 1881, p. 399.

(2.) "On Effects of Retentiveness in the Magnetisation of Iron and Steel," Proc. Roy. Soc., No. 220, 1882, p. 39.

(3.) "On the Magnetic Susceptibility and Retentiveness of Iron and Steel," Brit. Ass. Rep., 1883, and Phil. Mag., Nov., 1883. And an intimately related investigation is described in a paper "On the Production of Transient Electric Currents in Iron and Steel Conductors by Twisting them when Magnetised, or by Magnetising them when Twisted," Proc. Roy. Soc., No. 228, 1883, p. 117.

The effect of stress on thermoelectric quality was the first of these subjects to engage my attention.* In testing the changes of thermoelectric quality which a stretched iron wire underwent when successively loaded and unloaded so as to suffer alternate application and removal of tensile stress, I found that during increment and decrement of the load equal values of load were associated with widely different values of thermoelectric quality;† the difference being mainly of this character, that the changes of thermoelectric quality lagged behind the changes of stress. This lagging is, however, a *static* phenomenon, for it is sensibly unaffected by the speed at which the load is changed; and again, when any state of load is maintained constant the thermoelectric quality does not change with lapse of time. I then proceeded to search for other instances of the same kind of lagging, in other physical effects of stress, and also in effects caused by the change of other physical conditions besides stress. Magnetic phenomena present many instances of a similar action—some of which will be described below. Thus, when a magnetised piece of iron is alternately subjected to pull and relaxation of pull sufficiently often to make the magnetic changes cyclic, these lag behind the changes of stress in much the same way as the changes of thermoelectric quality do. And more generally, if a piece of iron, whether magnetised or not, be subjected to cyclic stress variations, it can be shown that certain of its physical qualities, while varying cyclically in consequence of the changes of stress, exhibit this same lagging. I found it convenient to have a name for this peculiar action, and accordingly called it Hysterēsis (from *ὑστερέω*, to lag behind).‡ Thus, when there are two qualities M and N such that cyclic variations of N cause cyclic variations of M, then if the changes of M lag behind those of N, we may say that there is hysteresis in the relation of M to N. The value of M at any point of the operation depends not only on the actual value of N, but on all the preceding changes (and particularly on the immediately preceding changes) of N, and by properly manipulating those changes, any value of M within more or less wide limits may be found associated with a given value of N.

§ 2. The effects of stress on magnetism and on thermoelectric quality will be recurred to at length further on. Meanwhile I shall describe experiments which were begun with the view of searching for hysteresis in the relation of magnetisation to magnetising force, without reference to stress.§ The presence of this peculiar action in the several parts of these researches forms a connecting link between them, but other lines of inquiry which suggested themselves in the progress of the experiments have also been followed up at considerable length.

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* 'Proceedings of the Royal Society,' No. 214, 1881, p. 399.

† This observation had, I afterwards learnt, been previously made by E. COHN (Wied. Ann., VI., p. 385).

‡ 'Proceedings of the Royal Society,' No. 216, 1881, p. 22, and No. 228, 1883, p. 123.

§ The results to be described below were all obtained before I became acquainted with the recent work of WARBURG on the same subject (Wied. Ann., XIII., p. 141), or with the less closely related observations of FROMME and AUERBACH, to which fuller reference will be made later (§ 32 below).

The fact that iron possesses retentiveness, or, in other words, that it remains magnetic after a magnetising force has been applied and removed, is of itself sufficient proof that there must be hysteresis in the relation of magnetism to magnetising force, when the changes of the latter are such that forces of opposite signs are alternately applied. But it does not follow that hysteresis is necessarily present when the changes of magnetising force are restricted to one direction and to one sign. The case may be supposed similar to that of a strained solid. When a stress along any axis is alternately applied and *reversed*, we know that (provided the resulting strain exceeds the body's limit of elasticity) there is hysteresis, of a simple and obvious character, in the relation of strain to stress. But if, instead of being reversed, the stress is merely applied and removed periodically, the greater part, if not the whole of the resulting change of strain (after the first application) is purely elastic, and we have no ground for asserting that there will then be hysteresis in the relation of strain to stress. In fact, if the process of magnetisation is strictly analogous to the straining of a solid whose limits of elasticity are exceeded by the strain, we should expect to find little or no hysteresis in the changes of magnetism which occur when after applying a magnetising force we (wholly or partially) remove and reapply it.

§ 3. The point under consideration has an important bearing on the theory of WEBER, which endeavours to explain the process of magnetic induction by supposing that the molecules of iron and other paramagnetic substances are always magnets, whose axes point indifferently in all directions, until, in consequence of applied magnetising force, they are turned more or less towards the direction in which the force acts. WEBER supposes that each magnetic molecule, when deflected from its initial position, tends to return to that position with a force which is the same as that which a magnetising force D, acting in the initial direction of its axis, would produce. If then deflection of the molecule be produced by the action of an applied magnetising force X, the direction which the molecule takes up while the force is at work is such that its magnetic axis points along the resultant of X and D. The force X exerts on it (per unit of its magnetic moment) a couple $X \sin \theta$, where θ is the angle made by its axis, in its deflected position, with the direction along which X acts. And the assumed restoring force D exerts a couple $D \sin \beta$, where β is the angle through which the molecule has been revolved. The molecule remains in equilibrium under these, and only these, forces, and when the magnetising force X ceases to act, it returns to its initial position through the action of D. Thus, the theory, in this form, takes no account of residual magnetism, and fails to explain retentiveness.

To remedy this defect, CLERK MAXWELL ('Electricity,' ii., Part 3, chap. vi.) has suggested a further assumption, based on the analogy of magnetism to mechanical strain. He supposes that when a molecule is deflected by a magnetising force X, it returns to its primitive position on the removal of X, provided the angle of deflection β has been less than a certain limit β_0; but if the deflection has exceeded β_0, then when X is removed the molecule does not completely return, but remains deflected through an angle $\beta-\beta_0$, which he calls the "permanent set" of the molecule. MAXWELL has developed the consequences of this supposition at length, but into these it is not at present necessary to follow him, further than to point out that (following up the analogy to a strained solid whose elasticity is perfect up to a certain limit, and which, when strained beyond that limit, acquires a new limit of elasticity at or near to the greatest value of the previously applied stress) we should expect to find, after the application of any force X, a purely elastic series of magnetic changes when we subsequently remove and reapply X. The movements of each molecule would then lie between the angle of previous permanent set $\beta-\beta_0$ and the angle β, and between those limits we should find each molecule taking up one and the same direction for each value of X, whether that was reached by increment from a lower, or decrement from a higher value. In other words, this modified theory leads us to expect no hysteresis in the relation of magnetic intensity to magnetising force when, after its first application, a magnetising force is subsequently removed and reapplied.

§ 4. To test this point, I have examined experimentally the cyclic changes of magnetisation which accompany cyclic changes of magnetising force, both when the force is confined to one direction, and also when its sign is reversed. The results do not support MAXWELL'S extension of WEBER'S theory, for they show that *all* changes of magnetism caused by changes of magnetising force exhibit that lagging action which I have called hysteresis. The word "retentiveness," commonly restricted to name that property in virtue of which the magnetic metals retain a portion of their magnetism when the magnetising force is removed, might with propriety be extended so as to designate the resistance to any change of magnetic state (whether increase or decrease) which they exhibit whenever the magnetic field in which they are placed suffers any change. The existence of residual magnetism when the field is reduced to zero is, in fact, only one case of an action which occurs whenever the field is varied in any way, namely, a tendency to persistence of previous magnetic state. And my experiments show that when the magnetism of iron is altered by varying, not the field, but the state of stress of the metal, the same tendency again manifests itself. The word retentiveness might be extended so far as to cover this case of lagging also; and if hysteresis were found only in these and similar magnetic phenomena, there would be no need to invent a name for it. But, as will be shown later, there are cases of hysteresis which have, as far as can be seen, nothing to do with magnetic condition, and which the quality of magnetic retentiveness, even if understood in the widest possible sense, cannot account for. I have therefore found it convenient and even necessary to employ a new term, which merely designates this peculiar action without implying any theory as to its cause.

§ 5. As regards the hysteresis which occurs when the magnetism of soft iron is changed, my experiments confirm the idea already suggested by other observers, that when the molecular magnets of WEBER are rotated they suffer, not first an elastic and then a partially non-elastic deflection as MAXWELL has assumed, but a kind of frictional retardation (resembling the friction of solids), which must be overcome by the magnetising force before deflection begins at all. Again, as the magnetising force begins to be withdrawn, this friction must be overcome by the quasi-elastic restoring force before each molecule can begin to return towards its primitive position; and the molecule finally takes up a position determined by the equilibrium of the restoring force and the frictional resistance to further return. It seems even possible that the residual magnetism which is found in soft iron when an applied magnetising force is completely removed, may be due entirely to this frictional sticking of the molecules. Hard iron and steel, on the other hand, in which the residual magnetism is more permanent, behave in a way which suggests the combination of this frictional sticking with something else of the nature of MAXWELL'S permanent set.

The frictional resistance must resemble the friction of solids, not the viscosity of liquids, since it appears to be independent of the speed at which the changes of magnetic condition occur; and since the magnetisation which any field produces is reached almost at once on the application of the field, and, so long as the field is kept constant, suffers little or no change with lapse of time.

§ 6. The following notation, adopted from MAXWELL, will be used throughout this paper :—

$\mathfrak{H}$ is the magnetising force.

$\mathfrak{B}$ is the magnetic induction within the metal. It is the number of "lines of force" per square centimetre which would be found if we were to cut an indefinitely narrow crevasse perpendicular to the direction of magnetisation.

$\mathfrak{I}$ is the intensity of magnetism, or the magnetic moment of the metal per cubic centimetre.

$\kappa, =\frac{\mathfrak{I}}{\mathfrak{H}}$, is the coefficient of induced magnetisation.

$\mu, =\frac{\mathfrak{B}}{\mathfrak{H}}$, is the magnetic permeability, or magnetic inductive capacity.

$$\mathfrak{B}=4\pi\mathfrak{I}+\mathfrak{H}$$
$$\mu=4\pi\kappa+1$$

The values of $\mathfrak{H}$, $\mathfrak{B}$, and $\mathfrak{I}$ will be given in c.g.s. units.

In representing graphically the results of experiments, I have generally plotted either $\mathfrak{B}$ or $\mathfrak{I}$ in terms of $\mathfrak{H}$. This is the most direct and, for most purposes, the most useful mode of representation. Another plan, used with good effect by ROWLAND, is to plot either κ or μ in terms of $\mathfrak{I}$ or $\mathfrak{B}$, and this has been adopted in a few instances.

§ 7. When a bar of finite length is magnetised by the action of an electric current in a surrounding solenoid, the magnetising field is due partly to the solenoid and partly to the ends of the magnet itself. During magnetisation the field due to the ends of the magnet opposes the action of the solenoid and reduces the resultant field, by amounts which vary from point to point along the axis of the bar, while after the magnetising current is reduced to zero there remains a demagnetising field, also unequal along the bar, due to the magnet itself. The magnetism which remains is therefore only that part of the true residual magnetism which is not removed by this self-demagnetising force: by "true residual magnetism" is meant the magnetism which would be found if after being magnetised the metal were left entirely free from all magnetising force. The influence of the ends has been recognised by many writers. To secure uniform magnetisation VON QUINTUS ICILIUS experimented with ellipsoids of revolution in place of cylindrical bars. STOLETOW* and ROWLAND,† by using endless magnets, have given their determinations of magnetic susceptibility and the maximum of magnetisation a value incomparably greater than that possessed by previous observations made on short bars. It is clear that the question which my experiments were, in the first instance, specially directed to examine, namely, whether hysteresis occurs in the relation of magnetism to field when the field is varied *without change of sign* (§ 2), could not be answered by any experiments with short bars, since in these the ends produce a magnetising force of reversed sign when the external force due to the solenoid is withdrawn. To answer this question we must use methods of experiment of the same rigorous character as are necessary in examining magnetic permeability or the coefficient of induced magnetisation.

To eliminate the action of the ends, and so secure the conditions suitable for exact experiment in the relation of $\mathfrak{H}$ to $\mathfrak{I}$, STOLETOW (on the suggestion of KIRCHOFF) and ROWLAND, used magnets whose form was that of a closed ring, and ROWLAND also used rods so long that the influence of the ends became negligible. With closed rings, as he has remarked, we can do no more than observe sudden changes in magnetism by using an induction coil, wound on the ring, in circuit with a ballistic galvanometer. With rods, on the other hand, we can examine the actual magnetic state at any instant, either ballistically, by drawing off a short induction coil from the middle portion of the length, or by direct magnetometric measurement. My own observations show that it is only when the length of the rod (if of iron) is about 300 or 400 times its diameter that the effect of length becomes insensible. The principal drawback to the use of very long rods is the difficulty of forming them (when the section is at all considerable) so as to be of sufficient length and at the same time homogeneous and uniform in section. By using a ballistic galvanometer whose needle, although sufficiently heavy to act ballistically, was comparatively light, I have been able to obtain good results with rods of very small section—*wires* in fact—which by the operation of drawing are scarcely less uniform in section than turned rods, and are in

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* Phil. Mag., xlv. (1873), p. 40.

† Phil. Mag., xlvi. (1873), p. 140, and xlviii. (1874), p. 321. An admirable summary of the work of STOLETOW and ROWLAND, as well as the earlier researches of WIEDEMANN, VON QUINTUS ICILIUS, THALÉN, and others, and the later contributions of many other observers, has been lately published by Professor CHRYSTAL in the article "Magnetism," Enc. Brit., ninth edition, vol. xv.

general much more homogeneous. Their small diameter also permits a solenoid of many turns to be wound on them without introducing much resistance, so that a strong magnetising force is obtainable without difficulty. With the single exception of an observation of cast-iron (§ 24 below), all the experiments to be described were made with wires. In some cases the wire has been bent into a ring and welded to form an endless magnet, but generally I have used long straight wires, which, as regards convenience of winding and other particulars, have many practical advantages over rings. A good form of magnet, especially appropriate in the examination of cast-iron and cobalt (where very long rods are impracticable), would be a nearly closed ring, with an opening between the ends just wide enough to allow a short induction coil to be slipped off. So far as the magnetisation of the middle portion was concerned, this would be nearly equivalent to a closed ring, while it would enable the actual magnetic state, and not sudden changes merely, to be measured.

§ 8. *Ballistic Method of Experiment.*—In the ballistic method the changes produced by sudden changes of $\mathfrak{H}$ were determined by the throw of the needle of a galvanometer in circuit with an induction coil which was wound over the middle part of the rod, or over the whole or any convenient part of the ring. The galvanometer was a short coil THOMSON's, made for lecture purposes, and consequently with an unusually heavy mirror. Its period was moderately long, and the retardation, although sufficient to bring the needle to rest without special appliances, in the interval between observations, was not so great as to unfit it for ballistic use. To test its accuracy in this kind of work a preliminary experiment was made, in which a wire in circuit with the galvanometer was wound into a coil of ten turns, which was slipped over the end of a long permanent bar-magnet. Readings were taken of the throws given when this coil was suddenly drawn off the bar, and again, at each stage, while the number of turns in the coil was progressively reduced to one. The throws were found to be proportional to the number of turns, to a degree of accuracy as great as was attainable in the actual observations, and throughout the whole range of the scale.

To reduce the ballistic readings to absolute measure, I adopted the method used by ROWLAND.[*] A coil of ten turns of thick wire, which will be called the earth-coil, having a total area of 10×1216 square centimetres, was wound on a light rectangular wooden frame, and was kept always in circuit with the ballistic galvanometer and the induction coil. It was laid flat on a horizontal table, and at the end of each experiment it was quickly turned over, thereby cutting twice the vertical component of the earth's magnetic force, and the throw of the galvanometer was observed. This "earth-coil reading" was the throw corresponding to $2 \times 12160 \times 0{\cdot}34$ lines of induction, 0·34 being the assumed local value[†] of the vertical component of the earth's force.

§ 9. Magnetising force was given by the current from a battery of gravity DANIELL

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[*] *Loc. cit.*, p. 148.

[†] In the Physical Laboratory of the University of Tokio, Japan, where the experiments were made.

cells, through a solenoid of moderately fine wire wound closely round the rod or ring. Even when the solenoid consisted of more than one layer, as it frequently did, the correction for the air space in evaluating 𝔅 was negligible, unless ℌ was unusually high. The force was varied, in the ballistic observations, by having in circuit with the magnetising solenoid a set of resistance coils which could be plugged up or inserted step by step so as to give a series of sudden increments or decrements in the value of ℌ. The magnetising current was measured by another mirror galvanometer, very strongly shunted, and its value in absolute measure was determined by seeing what deflection a given number of gravity cells in good condition gave through a given total resistance. In reducing the observations I have assumed that a gravity DANIELL cell in good condition, when put in circuit with a total resistance of n British Association units gives a current of $\frac{1{\cdot}1}{n}$ ampères. This agrees well with Dr. C. R. A. WRIGHT's recent determination;* and an experiment of my own, in which the current from one cell was adjusted by a variable resistance until it just sufficed to neutralise, in a vertical solenoid, the vertical component of the earth's force, gave a closely accordant value.

To have aimed at any very high degree of exactness in the reduction of magnetisation and magnetising force to absolute measure would have been a waste of time. The magnetic differences between different specimens of iron are so great that no useful purpose is served in determining absolutely the magnetic quality of any one specimen with extreme precision. On the other hand, even a somewhat rough reduction to absolute measure is so exceedingly serviceable in making results intelligible and definite in themselves, as well as capable of comparison with the work of other observers, as to justify fully the very considerable amount of arithmetical labour it involves. In dealing with measurements in which comparison of results was to be made, as, for instance, in the magnetisation of the same piece in different states of stress, I have been careful to make the reduced values accurately comparable with each other.

§ 10. *Magnetisation of an Iron Ring.*—In the following experiment a welded ring of moderately soft iron wire was subjected to a magnetising force, which was applied by a series of sudden steps up to 9·14 c.g.s. units, then reduced by steps to 0, and then reapplied, also by steps—with the view of testing the question raised in § 2, as to whether hysteresis occurs in the relation of 𝔅 to ℌ during the removal and reapplication of magnetising force. In this, the first example to be quoted, it may be well to give the numerical data and to exhibit the reduction in full: in subsequent examples a graphic representation of the reduced results, with occasionally a numerical statement of them, will suffice.

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* Phil. Mag. xiii. (1882), p. 265.

Diameter of wire.	0·248	centim.
Mean circumference of ring	31·4	centims.
Number of turns in magnetising coil	474	
Number of turns in induction coil	167	
Deflection of battery galvanometer with 3 cells and 6·85 ohms total resistance.	362	
Earth-coil reading	42·9	
Area of earth-coil	1216	sq. centims.
Number of turns in earth-coil	10	
Earth's vertical force	0·34	

Hence magnetising force for one scale division of the battery galvanometer

$$=\frac{4\pi\times3\times1\cdot1\times10^8\times474}{6\cdot85\times10^9\times362\times31\cdot4}=0\cdot02525.$$

And value of $\mathfrak{B}$ for one scale division of the ballistic galvanometer

$$=\frac{1216\times10\times2\times0\cdot34}{167\times42\cdot9\times\frac{\pi(0\cdot248)^2}{4}}=23\cdot89.$$

In the following table the actual readings are given, and also the values of $\mathfrak{H}$, $\mathfrak{B}$, $\mathfrak{I}$ and κ calculated from them. The ballistic readings at the beginning of the magnetisation are too small to allow κ to be determined for the early points with any approach to accuracy, and its early values are therefore omitted. It has been assumed that the ring was initially free from magnetism, which can have been no more than approximately true, for it had been moved about in the terrestrial field before the readings were taken.

The same experiment is shown graphically in Plate 57, fig. 1. It will be observed that although the changes of magnetism which occur during the removal and re-application of $\mathfrak{H}$ are small compared with the whole magnetisation, they exhibit hysteresis very decidedly with respect to their argument $\mathfrak{H}$.

Moderately soft iron ring, Plate 57, fig. 1.

Battery galvanometer reading.	$\mathfrak{H}$.	Ballistic throw.	Sum of throws.	$\mathfrak{B}$.	$\mathfrak{I}$.	κ.
5·3	0·13	1·1	1·1	26	2·1	
10·2	0·26	1·1	2·2	53	4·2	
12·0	0·30	0·5	2·7	65	5·1	
16·0	0·40	0·8	3·5	84	6·7	
21	0·53	1·0	4·5	107	8·6	
28	0·71	2·1	6·6	158	12·5	
37	0·93	2·9	9·5	227	18·0	
52	1·31	3·9	13·4	320	25·4	19
67	1·69	9·2	22·6	540	42·9	25
75	1·89	6·9	29·5	705	56·0	30
110	2·78	77·5	107·0	2,560	203	73
133	3·36	78·7	185·7	4,440	353	105
159	4·01	82	267·7	6,400	509	127
196	4·95	91·5	359·2	8,580	683	138
232	5·86	57	416·2	9,940	791	135
285	7·20	57	473·2	11,300	899	125
321	8·10	23·5	496·7	11,870	944	116
362	9·14	24	520·7	12,440	989	108
310	7·83	− 4·4	516·3	12,330	981	
246	6·21	− 6·7	509·6	12,170	968	
188	4·75	− 7·1	502·5	12,000	955	
107	2·70	−14·0	488·5	11,670	929	
0	0	−33·2	455·3	10,880	866	
110	2·78	15	470·3	11,240	894	
196	4·95	14·2	484·5	11,570	921	
246	6·21	11·9	496·4	11,860	943	
317	8·00	14·5	510·9	12,170	971	
362	9·14	10	520·9	12,440	990	

§ 11. Plate 57, fig. 2, shows the changes of magnetism of another ring, of very soft iron wire, when the magnetising force was (1) raised to 7·07, (2) reversed to −7·07, (3) restored to +7·07, (4) reduced to 0, (5) restored again to +7·07, all by a series of numerous sudden steps. Also, during the second operation two small loops were formed by reducing the force by steps to zero, reapplying it, and continuing the operation. In the curve of first magnetisation the greatest value of κ is 180, corresponding to $\mathfrak{H}=3$. During the reversal, as soon as the magnetising force changes its sign a very rapid demagnetisation begins, and a force of about −1·9 suffices to remove completely the residual magnetism. The reversal curves are steepest when the magnetisation is zero: at that point the value of $\frac{d\mathfrak{B}}{d\mathfrak{H}}$ is about 14,500. That is to say, for every line of force (per sq. centim.) which is then entering or leaving the field, 14,500 lines of induction are entering or leaving the substance of the metal. Complete double reversal has the effect of slightly reducing $\mathfrak{B}$ below the value reached in the first application of $\mathfrak{H}$, but removal and reapplication of the force raises $\mathfrak{B}$ slightly above its primitive value. Every loop in the diagram shows that when we reverse the *change* of magnetising force from increment to decrement, or *vice versâ*, the magnetisation begins to change very gradually relatively to the change of $\mathfrak{H}$, no matter how fast it may have been changing, in the opposite direction, before. So much is this the case that the curves, when drawn to a scale such as that of the figure, appear in all cases to start off tangent to a line parallel to the axis on which $\mathfrak{H}$ is measured, whenever the change of $\mathfrak{H}$ is reversed in sign.

§ 12. In a large number of other experiments I have examined the effects of varying the field $\mathfrak{H}$ in all possible ways. The accompanying changes of $\mathfrak{B}$ always exhibit static hysteresis with respect to the variations of $\mathfrak{H}$. The curves connecting these quantities always form loops as in fig. 2, and the characteristic mentioned in the last sentence of § 11 appears to be quite general. It is scarcely necessary to point out that this is not what we should expect from the theory of retentiveness suggested by MAXWELL (§ 3). On the other hand, it is just what we should expect if we suppose that there is a static frictional resistance to the rotation of WEBER'S magnetic molecules.

An interesting point may be noticed with regard to the small loops, two of which are shown in the diagram (fig. 2). Suppose that either on the ascending or the descending branch of the main curve we begin to form such a loop by removing the magnetising force, taking as the starting point a place in the main curve such that when $\mathfrak{H}$ becomes zero $\mathfrak{B}$ is also zero. Evidently this could be done by selecting a suitable starting point either in the positive part of the ascending branch of the main curve or in the negative part of the descending branch. Then when $\mathfrak{H}$ is reduced to zero the piece combines entire freedom from magnetisation with absence of magnetising force, but its condition is widely different from that of a previously unmagnetised piece. In particular, it is unsymmetrical as regards susceptibility to magnetisation in the two longitudinal directions, being much more ready to take magnetism of an opposite sign from that which it last possessed than to take magnetism of the same sign. The curves of $\mathfrak{I}$ and $\mathfrak{H}$ in the two quadrants, if we imagine them both to be drawn, would meet in a sharp angle at the origin, instead of being continuous, as they would be in a previously unmagnetised specimen. This is indeed only one example, though a very striking one, of the fact that in consequence of hysteresis a condition of no magnetism, in a field of no force, is capable of being reached by many processes, some of which will leave the metal ready to show a startling want of neutrality when it is subsequently magnetised in one or the other direction.

§ 13. *Residual Magnetism in Soft Iron.*—A feature in these and other early experiments which caused me much surprise was the largeness of the residual magnetism. In fig. 1, 87 per cent., and in fig. 2, 81 per cent. of the total induced magnetism remains when the magnetising force has been completely removed; and in other examples I have found the residual magnetism of soft iron to be 90 and even 93 per cent. of the induced magnetism. It is generally stated in the best text-books that the magnetic condition of soft iron disappears almost wholly when the inducing field is withdrawn. This entirely erroneous opinion has probably been established by experiments on rods whose length was insufficiently great to prevent them from demagnetising themselves more or less completely. In fig. 2 we see how great is the effect of a small amount of reversed force in reducing the residual magnetism. To illustrate the extent of the self-demagnetising action which occurs in even a moderately long rod, we may take the following ideal case. Suppose that an infinitely long, straight, round rod of radius r has been uniformly magnetised in the direction of its length, and the magnetising force withdrawn, leaving it with a uniform residual magnetisation whose intensity is $\mathfrak{J}$. Then imagine a length $2a$ to be suddenly cut out of it, and the ends to be instantaneously removed. At the instant of separation the magnetisation may still be supposed to be uniform, and the self-demagnetising force which then acts at any point in the axis of the rod at a distance x from the centre is

$$-2\pi\mathfrak{J}\left\{2-\frac{a-x}{\sqrt{r^2+(a-x)^2}}-\frac{a+x}{\sqrt{r^2+(a+x)^2}}\right\}.$$

Such a distribution is of course self-destructive; but it is interesting to examine the numerical values of the demagnetising force it would cause, with a given value for $\mathfrak{J}$ and for the ratio of length $2a$ to radius r. Let the rod's length be fifty times its diameter, and suppose that it consists of the same soft iron as the ring of fig. 2, and has been similarly treated before cutting, so that the residual value of $\mathfrak{B}$ at the instant of cutting is 9000. The following numbers show the resulting values of the demagnetising force due to the ends at various points along the axis of the bar :—

Distance from centre expressed as a fraction of the length.	Self-demagnetising force in c.g.s. units.
0·5	4500
0·495	2488
0·49	1318
0·48	475
0·45	87
0·4	22·7
0·35	10·3
0·3	6·0
0·25	4·6
0·2	2·9
0	1·8

Comparing these with fig. 2 we see that throughout very nearly its whole length the rod would be subjected in this ideal case, by its own magnetism, to a force more than sufficient to remove that magnetism altogether. It need not therefore cause surprise that there is little residual magnetism in the equilibrium distribution which is arrived at after any applied magnetising force is withdrawn from a rod whose length is even as much as fifty times its diameter.

§ 14. Another cause contributes to produce the erroneous opinion that soft iron has little retentiveness, namely, the enormous influence which the slightest mechanical disturbance has in removing the residual magnetism. The soft iron ring of fig. 2 loses nearly all its residual magnetism if gently tapped after the magnetising force has ceased to act. I have frequently removed in this way all but one or at most two per cent. of the originally large residue. So susceptible indeed is soft iron to the effect of mechanical disturbance, that when the field is removed the lightest touch by the fingers suffices to destroy a large part of the residual magnetism, although it appears that so long as the iron is left perfectly undisturbed the residual magnetism does not suffer loss with lapse of time. I shall recur to the effects of vibration later; meanwhile they are mentioned in order to explain that we may take advantage of this property to reduce a piece of soft iron to a very nearly neutral state after magnetisation, so that we may study the comparative effects of again magnetising the same piece under different conditions. Simple tapping in a field of no force reduces the piece to a state differing very little from that in which it was after annealing and before any magnetisation had taken place. This I have found to be the case by taking curves of magnetisation of a very soft iron wire after annealing, and again after shaking out the residual magnetism: such curves turn out to be sensibly coincident.

§ 15. *Magnetisation of long Rods of various lengths.*—Taking advantage of this property, in virtue of which we can reduce a given specimen of soft iron after magnetisation to a state nearly identical with its primitive state, I made the following group of observations to illustrate the influence of length on the magnetisation of straight rods of circular section. A long straight wire of the same very soft iron as the ring of fig. 2 was well annealed, wound with a magnetising solenoid and placed on a horizontal table in the E.-W. position. After the wire was tapped to get rid of its initial magnetism a curve of $\mathfrak{B}$ and $\mathfrak{H}$ was taken by the ballistic method. $\mathfrak{H}$ was then reduced to zero and the residual magnetism was shaken out by tapping the wire on the table, still keeping the E.-W. position, and the permanent residue (which was very small) was determined by slipping off the induction coil. The induction coil was about 5 centims. long, and was placed at the centre of the rod's length. The process of magnetisation was repeated three times, giving sensibly the same results each time. The wire's diameter was 0·158 centim., and its original length was 47·5 centims., or 300 diameters. Then the length was reduced to 31·6 centims., or 200 diameters, by cutting off equal portions from both ends, and a curve of magnetisation was taken. The residual magnetism was again tapped out, the length again reduced in the same way to 150 diameters, and another curve of magnetisation taken. The same process was repeated with the length equal to 100, 75, and 50 diameters successively. The peculiarity of the method lay in this, that throughout the series of observations the magnetic induction was determined through precisely the same piece of material. The influence of length *per se* was therefore exhibited in a way which would have been impossible had the successive tests involved the unknown and not inconsiderable differences of magnetic quality which different specimens would have possessed or different annealings given.

In each case the magnetising force due to the solenoid was raised by steps to the same value, 34·2 c.g.s. units, and lowered by steps to zero. The curves of Plate 57, fig. 3, show its relation to 𝔅 at the centre of the rod in the various cases. The full lines are the "on" curves, or curves showing the relation of 𝔅 to the solenoid's magnetising force during its application: the dotted lines are the "off" curves, and show the values of 𝔅 during decrease of the solenoid's magnetising force. The number affixed to each curve gives the ratio of length to diameter in the test it belongs to. The abscissas give the magnetising force due to the solenoid, which of course differs from ℌ by the amount of the variable field which the rod exerts upon itself. To avoid confusion in the figure, a portion only of each pair of curves is drawn, except in the case of the longest and shortest rods, for which the curves are drawn in full. The greatest magnetisation reached was sensibly the same for the longest rods and for those of intermediate lengths. At the beginning of each "on" curve a small amount of initial magnetism will be noticed, which is the part of the previous residual magnetism not shaken out by tapping.

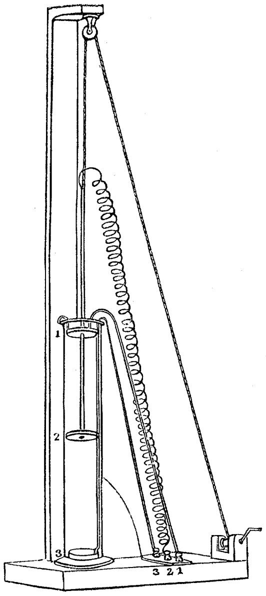
§ 16. It appears that the rod 300 diameters long differs little in its magnetic behaviour from an indefinitely long rod or from a ring. Its residual magnetism is 85 per cent. of the total, and its greatest value of μ is about 3500. The "on" curve of the 200 diameters rod falls not much below that of the 300 diameters rod, but its "off" curve is notably different near the conclusion, where the demagnetising influence of the ends becomes sharply apparent. The result is to reduce the residual magnetism to 60 per cent. of the total. In the 150 diameters rod the residual magnetism falls to 39 per cent., in the 100 diameters rod to 20 per cent., in the 75 diameters rod to 9 per cent., and in the 50 diameters rod to about 6 per cent. The "off" curves of the shorter rods are distinguished by a long straight descent towards the axis of 𝔅, showing a sensibly uniform rate of demagnetisation during the later part of the withdrawal of the externally applied force.

The comparative difficulty of magnetising a short rod is shown by fig. 3 in a way which requires no comment. If we were to infer values of μ and κ from the test of the 50 diameters rod, neglecting the fact that the magnetising force due to the solenoid is not the whole of ℌ, we should find for the maximum of μ 656, and for that of κ 52; and these coefficients are much more nearly uniform for various values of the magnetising force than when we determine them in a legitimate manner by observing the magnetisation of very long rods or of rings.

§ 17. From this and other experiments, I concluded that even when dealing with the softest iron, we may take a rod whose length is not less than 300 diameters as giving results scarcely different from those given by a ring or a longer rod, and in subsequent experiments rods were almost exclusively used whose length was from 300 to 400 times their diameter. In hard iron and in steel a smaller ratio of length to diameter would no doubt give an equally good approximation to the condition of endlessness.

It should not be forgotten that want of perfect homogeneity introduces a self-demagnetising force even in the longest rod or in a ring, by causing some of the lines of induction to escape from the substance of the metal at intermediate points, and it does not appear that we can easily, if at all, approximate more closely to the ideal condition of uniform magnetisation than by the use of a straight piece of wire in which the ratio of length to diameter is (say) 400. If it be desired to deal with a larger section of metal than a single wire, a good plan is to form a multiple ring by winding a long continuous wire into a coil of as many turns as may be wished, and cutting the ends so that they abut against one another. This gives a more homogeneous structure than is got by welding a rod into a ring, or even by turning a ring out of a solid forged piece. One or two experiments made with multiple rings have only served to confirm the general accuracy of the results got by the use of straight wires.

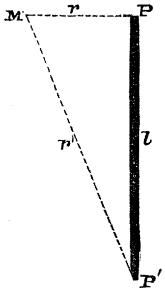
§ 18. *Direct Magnetometric Method of Experiment.*—In addition to the ballistic method, a direct magnetometric method has been employed in many cases in examining the magnetisation of long wire rods. The wire was placed in the vertical position, with its upper end nearly level with, and magnetically east of a small mirror magnetometer, formed by suspending the mirror of a THOMSON galvanometer by a silk fibre about 8 centims. long. Two coils were wound round the rod: through one of these a constant current was maintained which just sufficed to neutralise the vertical component of the earth's magnetic field. The other formed the magnetising solenoid, and the current in it was varied not by sudden steps as in the ballistic method, but



continuously by means of a "slide" of very simple construction, which is shown in the annexed figure. This consisted of a tall cylindrical glass jar filled with a dilute solution of sulphate of zinc, with a block of amalgamated zinc fixed at the top, and another at the bottom, the blocks being connected by the terminals 1 and 3 to the two poles of the battery. A third block of zinc was raised or lowered between these, through the fluid, by means of a cord and pulley, and formed the sliding terminal of the magnetising solenoid, of which one end was connected to it by the terminal 2, and the other end was connected through a commutator to one of the fixed blocks. This liquid slide gave a remarkably effective and convenient means of varying the magnetising current, and no difficulty was experienced in keeping the current constant when it was desired to do so.

The position of the magnetised wire with respect to the magnetometer was such that any change in *distribution* of magnetism produced minimum effect, the displacement of the upper "pole" being at right angles to the line joining it with the magnetometer, while the lower pole was comparatively inoperative. The reduction of the observations to absolute measure presents no novel features. $\mathfrak{I}$ was first evaluated, the deflection due to the magnetising solenoid alone being separately determined, and subtracted from the observed magnetometer readings before these were reduced to find $\mathfrak{I}$. $\mathfrak{B}$, where it is given, was afterwards calculated from $\mathfrak{I}$ and $\mathfrak{H}$.

Let PP′ be the magnetised wire, whose length is l and moment lm. Let M be



the position of the magnetometer, distant r from P and r' from P′, and let H be the horizontal component of the earth's field, acting perpendicular to the plane MPP′. Then if θ be the deflection of the magnetometer needle,

$$\mathrm{H} \sin \theta = \left(\frac{m}{r^2} - \frac{m}{r'^2} \cdot \frac{r}{r'}\right) \cos \theta,$$

$$m = \frac{\mathrm{H} r^2 \tan \theta}{1 - \left(\frac{r}{r'}\right)^3}.$$

Or, calling d the diameter of the wire,

$$\mathfrak{I} = \frac{4lm}{\pi d^2} = \frac{4l\mathrm{H}r^2 \tan \theta}{\pi d^2 \left\{1 - \left(\frac{r}{r'}\right)^3\right\}}.$$

In this simple form the direct magnetometric method, as a means of determining magnetic qualities in absolute measure, is not entirely free from objection, but it allows certain points to be investigated which the ballistic method is unable to deal with, notably the influence of time in magnetisation, and the comparative effects of sudden and gradual application or removal of magnetising force, and it is also particularly convenient in testing the effect of stress on magnetic susceptibility.

§ 19. *Demagnetisation by the method of reversals.*—As an alternative to the plan of tapping out the magnetism, which is inapplicable to steel or hard iron, another plan has more frequently been used to remove residual magnetism when it has been desired to repeat the magnetisation of a given specimen without change of its physical state. This consists in subjecting the piece to a succession of magnetising forces with opposite signs and of gradually diminishing intensity. By means of the liquid slide described above, the current was slowly reduced to zero, while a rapid reversing key between the slide and the solenoid was kept in action. The earth's vertical force was, as has been said, balanced by a steady current in a separate solenoid, and when the strength of this last was properly adjusted, the process of demagnetising by reversals was very successful in removing all traces of residual magnetism.* It is clear that this process leaves the piece symmetrically conditioned as regards subsequent magnetisation in either of the two longitudinal directions. The condition of a piece demagnetised in this way is no doubt different from that of a piece never before magnetised (indeed the curves of $\mathfrak{I}$ and $\mathfrak{H}$, taken before and after the process, show that it is somewhat, though not very, different as regards susceptibility); but the neutral state produced by this process forms an exceedingly convenient starting point, to which we may recur over and over again, when we wish to determine the susceptibility or the retentiveness of the same piece under different conditions of stress, temperature, &c.

§ 20. *Cyclic Magnetisation of Wrought-iron Wires.*—A few experiments will now be detailed in which well annealed† specimens of soft iron wire were subjected to a complete cycle of magnetisation by the successive application of equal and opposite magnetising forces. In each case the length of the specimen was so great as to realise, as nearly as possible, the condition of endlessness. In the following experiment, the results of which are given numerically below, and graphically in Plate 57, fig. 4, the specimen was a very soft iron wire 0·158 centim. in diameter, and 64 centims. or 400 diameters in length; and the method used was the step-by-step ballistic method (§ 8). The steps by which $\mathfrak{H}$ and $\mathfrak{B}$ were varied were those shown in the table below. The magnetising force was first raised to 17·26, then reversed

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* A process similar in principle was applied some years ago by Mr. H. S. MAXIM to cure magnetised watches.

† The process of annealing consisted in drawing the wire through a blowpipe flame so slowly that each part of the length in turn became bright red and then cooled gradually as it moved away from the flame. For wires of less than 1 mm. diameter an ordinary spirit-lamp flame was generally used, without a blowpipe.

to —17·26, reversed again to +17·26, reduced to 0, and restored to +17·26, all by steps. At the beginning of the operation the wire had a small amount of initial magnetism, which was determined by sliding off the induction coil.

VERY Soft Annealed Iron Wire, Plate 57, fig. 4.

ℌ.	𝔅.	ℌ.	𝔅.	ℌ.	𝔅.	ℌ.	𝔅.
0	166 (initial)						
0·05	166	— 0·05	10,970	0·05	—10,795	0·05	10,845
0·19	183	— 0·19	10,790	0·19	—10,620	0·19	10,850
0·37	216	— 0·37	10,460	0·37	—10,268	0·57	10,860
0·71	393	— 0·71	9,530	0·71	— 9,270	0·71	10,890
1·08	941	— 1·08	7,360	1·08	— 6,780	1·08	10,930
1·84	4,088	— 1·84	20	1·84	— 642	1·84	11,040
3·30	8,684	— 3·30	— 7,840	3·30	8,190	3·30	11,480
5·62	11,380	— 5·62	—11,100	5·62	11,270	5·62	12,210
7·63	12,320	— 7·63	—12,080	7·63	12,170	7·63	12,650
10·54	12,950	—10·54	—12,690	10·54	12,770	10·54	13,040
14·19	13,280	—14·19	—13,030	14·19	13,110	14·19	13,300
17·26	13,450	—17·26	—13,190	17·26	13,280	17·26	13,440
14·19	13,370	—14·19	—13,110	14·19	13,220		
10·54	13,250	—10·54	—13,000	10·54	13,100		
7·63	13,100	— 7·63	—12,900	7·63	12,940		
5·62	12,970	— 5·62	—12,800	5·62	12,830		
3·30	12,680	— 3·30	—12,520	3·30	12,570		
1·84	12,240	— 1·84	—12,080	1·84	12,150		
1·08	11,850	— 1·08	—11,700	1·08	11,790		
0·71	11,590	— 0·71	—11,430	0·71	11,510		
0·37	11,277	— 0·37	—11,130	0·37	11,200		
0·19	11,100	— 0·19	—10,970	0·19	11,030		
0·05	11,000	— 0·05	—10,820	0·05	10,880		
0	10,980	0	—10,800	0	10,840		

Here the residual magnetism is 82 per cent. of the induced magnetism, both on the positive and on the negative side. An opposite force of about —1·9 serves to remove it completely. The general form of the curves in fig. 4 should be noted, since, as many other experiments have shown, this is a thoroughly typical example of the behaviour of annealed iron.

§ 21. In the next example the method of experiment was different. The magnetising force was varied continuously and very slowly by means of the liquid slide described in § 18, occasional pauses being made to allow current and magnetometer observations to be taken. The magnetisation was measured by the direct magnetometric method of § 18. The wire was a piece of moderately soft iron, well annealed, of 0·077 centim. diameter, and 30·5 centims. or 400 diameters in length. The magnetising force was raised to 22·3, reversed to —23, and restored to +22·3, all very gradually. Whenever, as was frequently the case, the magnetometer deflection appeared to be creeping up or down after the alteration of magnetising force had ceased, time was given to allow the magnetisation to become sensibly steady before a reading was taken. The following corresponding values of ℌ and 𝔅 are reduced from the observations; they are also exhibited graphically in Plate 58, fig. 5.

ANNEALED Iron Wire, Plate 58, fig. 5.

Current in magnetising solenoid,—galvr. reading.	ℌ.	Magnetometer reading.	𝔅.	μ.	Current in magnetising solenoid,—galvr. reading.	ℌ.	Magnetometer reading.	𝔅.	Current in magnetising solenoid,—galvr. reading.	ℌ.	Magnetometer reading.	𝔅.
0	0	0	0									
9	0·32	1	41	128	11·5	− 0·41	342	14,140	9	0·32	−348	−14,390
24	0·85	4	165	194	23	− 0·81	329	13,600	19	0·67	−339	−14,010
39	1·38	10	413	299	31	− 1·10	318	13,150	29	1·02	−327	−13,520
59	2·18	28	1,460	670	41	− 1·45	295	12,200	39	1·38	−306	−12,650
79	2·80	89	3,680	1310	51	− 1·80	253	10,460	49	1·73	−274	−11,330
99	3·50	175	7,230	2070	62	− 2·20	166	6,860	59	2·09	−195	− 8,060
119	4·21	239	9,880	2350	71	− 2·51	70	2,890	69	2·44	−103	− 4,260
139	4·92	279	11,540	2350	81	− 2·87	− 12	− 496	79	2·80	+ 4	165
159	5·63	304	12,570	2230	91	− 3·22	− 83	− 3,430	86	3·04	58	2,400
189	6·69	327	13,520	2020	101	− 3·50	−142	− 5,870	99	3·50	130	5,370
239	8·46	348	14,390	1700	121	− 4·28	−226	− 9,340	114	4·03	199	8,230
289	10·23	359	14,840	1450	141	− 4·99	−278	−11,490	140	4·96	279	11,530
342	12·11	365	15,090	1250	162	− 5·73	−306	−12,650	190	6·72	332	13,720
441	15·61	373	15,420	990	211	− 7·47	−338	−13,970	240	8·50	351	14,510
574	20·32	378	15,630	770	261	− 9·23	−352	−14,550	291	10·30	362	14,970
629	22·27	380	15,710	705	312	−11·05	−361	−14,920	339	12·00	368	15,210
464	16·42	379	15,670		411	−14·55	−369	−15,250	340	12·04	374	15,460
239	8·46	375	15,500		511	−18·09	−373	−15,420	579	20·50	381	15,750
139	4·92	372	15,380		652	−23·08	−376	−15,550	630	22·30	383	15,830
89	3·15	369	15,270		536	−18·94	−376	−15,550	0	0	353	14,590
39	1·38	363	15,010		411	−14·55	−376	−15,550				
0	0	350	14,470		311	−11·01	−375·5	−15,530				
					211	− 7·47	−375	−15,500				
					111	− 3·93	−372	−15,380				
					61	− 2·16	−368	−15,210				
					23	− 0·81	−361	−14,930				
					11·5	− 0·41	−358	−14,800				
					0	0	−352	−14,550				

The figures show that the wire of this experiment, although less susceptible than the last to low values of the magnetising force, took a greater magnetisation with high values of the force. But the most notable feature of this experiment is the enormously great amount of residual magnetism which this wire retained when the magnetising force was completely withdrawn. On the positive side the residual was 92 per cent. of the induced magnetism, and on the negative side 92·5 per cent. Its absolute value is very remarkable. 𝔅=14,500 (the residual value) corresponds to an intensity of magnetism (ℑ) of 1150 c.g.s. units of moment per unit of volume. Great as this is, it was even slightly exceeded in another experiment with the same wire, in which 93 per cent. of the induced magnetism remained after the magnetising force was removed, giving a residual intensity of 1200 c.g.s. units of moment per unit of volume. So far as I am aware no steel magnet ever holds so much magnetism as this. In fact, if by retentiveness we mean simply the faculty of remaining magnetic when magnetising force is removed, and in the absence of mechanical or other disturbance, soft iron is far more retentive than either hard iron or steel.

§ 22. One more example may be given of the application of a complete magnetising cycle to annealed iron. In this a much stronger magnetising force was used than before. The wire was a piece nearly the same as that of the last experiment, diameter 0·078 centim., length 29·3 centims., or nearly 400 diameters. The observations (which were made by the direct magnetometric method) are given in a reduced form below, and are also exhibited in Plate 58, fig. 6.

ANNEALED Iron Wire, Plate 58, fig. 6.

$\mathfrak{H}$.	$\mathfrak{B}$.	μ.	$\mathfrak{H}$.	$\mathfrak{B}$.	$\mathfrak{H}$.	$\mathfrak{B}$.
0	800 (initial)	—				
2·8	4,830	1725	− 4·35	− 6,300	4·7	11,130
3·7	7,510	2030	− 7·8	−11,540	7·1	11,940
4·8	9,520	1980	−15·5	−14,360	11·7	13,690
6·2	11,130	1795	−46·6	−15,200	15·5	14,360
7·8	12,340	1580	−89·8	−16,040	23·3	14,770
9·3	13,140	1410	−26.7	−14,910	38·8	15,590
10·9	13,550	1240	−19·5	−14,630	66·8	16,160
14·6	14,220	970	−15·5	−14,490	89·8	16,310
23·3	14,900	640	−11·2	−14,360		
31·1	15,180	490	− 7·9	−14,080		
46·6	15,700	340	− 6·2	−13,950		
69.9	16,160	230	− 4·6	−13,680		
89·8	16,450	183	− 4·0	−13,410		
0	12,740		0	−12,070		
15·5	14,760		1·5	− 8,580		
31·0	15,310		2·4	− 2,680		
62·1	16,010		2·9	670		
89·9	16,450		3·1	1,880		
62·1	16,150		3·6	4,020		
31·0	15,480		3·7	4,830		
23·3	15,310		4·3	6,970		
15·5	15,030		4·8	8,320		
10·9	14,890		5·1	8,850		
7·8	14,620		5·4	9,390		
4·7	14,480		7·1*	12,080		
0	12,740		6·1	11,940		
			4·6	11,800		
			2·5	11,400		
			0	10,460		

§ 23. *Characteristics of the Curve of Magnetisation.*—The examples which have been cited exhibit very clearly the behaviour of annealed iron when subjected to magnetising forces without mechanical disturbance. It matters little whether the changes of magnetising force are caused to occur suddenly or gradually: there are slight differences in the two cases which will be discussed later, but the general form of the curve is scarcely changed. A comparison of the figures already given will show that the curve of first magnetisation has the following characteristics. At first the magnetisation takes place very gradually with respect to the increment of $\mathfrak{H}$,—how gradually these experiments do not enable me to say, but the appearance of the

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* A small loop was formed here by reducing $\mathfrak{H}$ from 7·1 to zero, and reapplying it. See the figure.

curves is not inconsistent with the idea that $\frac{d\mathfrak{I}}{d\mathfrak{H}}$ is initially zero. By the time the magnetising force reaches the value of 1 c.g.s. unit or so, a pretty sharp upward bend takes place, and the magnetisation proceeds rapidly and at a nearly uniform rate for a considerable length of the curve. Then this rate changes gradually, as the region of what is popularly known as saturation is approached, in which region we again find a nearly straight portion of the curve, where a continuous though much less rapid increase of magnetism is still taking place. With magnetising fields whose strength is 100 units or so (my experiments did not deal with stronger fields) the values both of $\mathfrak{B}$ and of $\mathfrak{I}$ show no sign of reaching a limit.

During the withdrawal of the magnetising force, if that has been strong, the magnetism stays well up in the region of so-called saturation until the force is almost wholly removed. As soon as a small force of opposite sign is applied the residual magnetism begins to disappear, and opposite magnetism to take its place, with great rapidity. The rate of this change with respect to the change of $\mathfrak{H}$ is however very uniform during the greater part of its progress. Other characteristics, such as the hysteresis which occurs when a magnetising force is removed and reapplied, have been already described, and others still will be evident on inspection of the curves.

One feature which deserves special notice is the large amount of magnetism which even a very weak field will induce in annealed iron, when the specimen is endless, or long enough to prevent its ends from materially modifying the field. Take for instance the experiment last cited. With a field of 90 c.g.s. units the value of the induction $\mathfrak{B}$ is 16,500, but to produce an induction of 10,000 a field of less than 5 units is sufficient. And this susceptibility, high as it is, is greatly exceeded when during the application of the magnetising force the iron is subjected to mechanical vibration (see § 50).

The curves showing the relation of magnetisation to magnetising force have a very different form when we are dealing with hard-drawn iron, or with a sample which instead of being in the annealed state, has been strained beyond its limit of elasticity before being magnetised. Examples of these will be given later ; meanwhile the following experiments with cast-iron and steel may be placed here for convenience of comparison with soft iron.

§ 24. *Magnetisation of a Cast-iron Ring.*—The ring was turned circular, and of circular section, the dimensions being—

	Centims.
External diameter	15·05
Internal ,, ...	12·30
Diameter of section	1·378

The method of experiment was the step-by-step ballistic method of § 8. The results are shown in Plate 58, fig. 7. A large loop was formed by applying and reversing and reapplying a magnetising force of nearly 16 units ; and then a small loop was formed by removing and reapplying the same force. This force was insufficient to make the cast-iron approach magnetic saturation. The numerical values of $\mathfrak{H}$ and $\mathfrak{B}$ are given below: the initial value of $\mathfrak{B}$ could not of course be determined, and it is assumed to have been zero. [Owing to an omission in the laboratory note-book some uncertainty attaches to the reduction of the magnetism of this ring to absolute measure. The example serves, however, to show the presence of hysteresis in the magnetisation of this material.]

CAST-IRON Ring, Plate 58, fig. 7.

$\mathfrak{H}$.	$\mathfrak{B}$.	$\mathfrak{H}$.	$\mathfrak{B}$.	$\mathfrak{H}$.	$\mathfrak{B}$.
0	0	− 0·73	2450	0·67	−2100
0·07	3	− 0·95	2420	0·95	−2050
0·20	12	− 1·26	2380	1·50	−1960
0·38	24	− 1·87	2290	2·39	−1790
0·73	48	− 2·37	2200	3·27	−1530
1·13	76	− 3·27	2010	3·94	−1390
1·87	145	− 3·94	1860	4·76	−1080
3·27	344	− 4·76	1600	5·77	− 620
5·54	864	− 5·74	1210	7·34	320
7·34	1490	− 7·34	360	8·86	1350
9·94	2360	− 8·86	− 660	11·28	2390
13·15	3170	−11·28	−1990	13·15	2895
15·75	3680	−13·15	−2670	15·75	3450
13·15	3560	−15·71	−3420	13·15	3330
9·81	3390	−13·15	−3300	9·94	3150
7·34	3200	− 9·94	−3200	7·34	2980
5·41	3080	− 7·34	−2940	5·41	2830
3·27	2900	− 5·41	−2790	3·27	2640
5·83	2760	− 3·27	−2590	1·83	2500
1·13	2680	− 1·87	−2430	1·13	2410
0·73	2640	− 1·26	−2340	0	2280
0	2550	− 0·73	−2320	1·13	2340
		0	−2200	1·83	2370
				3·27	2460
				5·41	2590
				7·34	2710
				9·93	2920
				13·13	3190
				15·71	3460

§ 25. *Magnetisation of Steel Wire in the hard-drawn, annealed, and glass-hard states.*—The next three experiments were made on a piece of steel wire 0·137 centim. in diameter, and 19·7 centims. or 144 diameters in length. This length would have been insufficient to allow of a fair experiment if the material had been soft iron, but steel is so much less susceptible that the effect of the ends is less important, and a condition approaching that of endlessness is reached with a shorter length than would be admissible in the case of iron. In all three experiments the step-by-step ballistic method was used.

The wire was first tested when in what may be called its normal temper, the hard-drawn state in which it was commercially supplied. Plate 58, fig. 8, shows its behaviour when a magnetising force of 57·5 units was applied, reversed, reapplied, removed and reapplied. The greatest value of $\mathfrak{B}$ reached was between 14,000 and 15,000, and of this 57 per cent. survived the removal of the inducing force.

§ 26. The same wire was then annealed by heating to bright redness and cooling slowly, and observations made on it whose results are given graphically in Plate 58, fig. 9. The curves here have a form much more similar than those of fig. 8 to the curves for soft iron, but the susceptibility to moderate magnetising forces is much less than in iron. The induced magnetism is nearly the same as for the hard-drawn wire, but the residual magnetism is considerably greater for the annealed than for the hard-drawn wire, being now from 76 to 80 per cent.

§ 27. In the next experiment the same piece of wire was again tested, after being made glass-hard by being plunged into water while at a bright red heat. Its magnetic behaviour in this state is shown in Plate 58, fig. 10. The highest magnetising force applied (over 55 c.g.s. units) did not carry the wire past the steep part of the curve of magnetisation, and gave only 9300 as the value of $\mathfrak{B}$. On the removal of this force the residual value was 6360, or 68 per cent. of the induced. This must not, however, be taken as a measure of the percentage of residual magnetism which would have been found had the steel been more nearly "saturated." To remove the residual magnetism a reverse force of over 40 units was necessary. A noticeable feature in this case is the smallness of the hysteresis when the magnetising force was withdrawn and reapplied. The "off" and "on" curves for these operations are almost coincident. In fact, so far as this point is concerned, MAXWELL's extension of WEBER's theory (§ 3) would serve satisfactorily enough to explain the magnetic retentiveness of this material. Figs. 8, 9 and 10 are all drawn to the same scale, which is also the scale of fig. 6.

§ 28. *Experiments with Pianoforte Steel.*—The foregoing experiments on steel showed the desirability of testing that material under greater values of the magnetising force. Accordingly another series were made in which the method used was the direct magnetometric method (§ 18), and the magnetising force was raised to over 90 units. The wire used in this series of tests was pianoforte steel 0·078 centim. in diameter, and 30 centims. or 380 diameters long. Observations were first made with the wire in its normal commercial temper; their results are given below and also in Plate 58, fig. 11, which shows the relation of $\mathfrak{B}$ to $\mathfrak{H}$ while the magnetising force was being applied, reversed, reapplied, removed, and reapplied as in former examples.

PIANOFORTE Steel Wire, Normal Temper, Plate 58, fig. 11.

$\mathfrak{H}$.	$\mathfrak{B}$.	μ.	$\mathfrak{H}$.	$\mathfrak{B}$.	$\mathfrak{H}$.	$\mathfrak{B}$.	$\mathfrak{H}$.	$\mathfrak{B}$.
0	76		− 4·37	11,510	27·30	+3,590	+ 7·80	12,020
4·40	230		− 7·80	10,850	31·20	4,300	15·60	12,240
7·80	460	59	−15·60	8,580	35·20	7,830	31·20	12,690
15·60	1,540	99	−23·40	3,380	39·16	10,190	39·00	12,950
23·40	4,320	185	−31·20	− 4,115	46·80	12,080	46·80	13,230
31·20	7,770	249	−39·00	− 9,810	54·60	12,950	62·40	13,720
39·00	10,630	273	−46·80	−12,000	62·56	13,490	78·16	14,170
46·88	12,110	258	−62·48	−13,520	78·00	14,170	91·89	14,510
54·60	12,920	237	−78·00	−14,220	92·20	14,590		
62·40	13,490	216	−91·57	−14,590				
70·20	13,890	198			70·04	14,360		
78·00	14,200	182	−70·20	−14,320	46·80	13,820		
91·66	14,590	159	−46·80	−14,010	31·20	13,330		
			−31·20	−13,520	15·60	12,840		
70·20	14,300		−15·60	−12,950	7·80	12,400		
46·80	13,840		− 7·80	−12,550	0	11,850		
31·20	13,440		0	−11,970				
15·60	12,870		+ 7·80	−11,070				
7·80	12,420		15·60	− 9,020				
0	11,850		23·40	− 3,250				

§ 29. Plate 59, fig. 12 shows the behaviour, under corresponding variations of magnetising force, of a piece of pianoforte steel wire of the same quality and dimensions as the above, after it had been annealed by heating to bright redness and cooling slowly. The following figures apply to the first application of $\mathfrak{H}$:—

PIANOFORTE Steel Wire, Annealed, Plate 59, fig. 12.

$\mathfrak{H}$.	$\mathfrak{B}$.	μ.
0	−31 (initial)	—
4·99	398	80
7·80	895	115
11·70	2,010	172
15·66	3,585	229
19·50	5,220	268
23·40	6,750	289
27·30	8,040	295
31·20	9,090	291
35·26	9,900	281
39·00	10,530	270
43·06	11,110	258
46·88	11,520	246
54·60	12,300	225
62·56	12,920	207
70·28	13,390	191
78·47	13,850	176
91·73	14,400	157

§ 30. Plate 50, fig. 13, shows the behaviour of a third piece of pianoforte steel wire cut from the same bundle as the two last, but rendered glass-hard by sudden cooling in water. This time the length was 22 centims. or nearly 300 diameters. The numerical values of $\mathfrak{H}$, $\mathfrak{B}$, and μ during the first application of the force are given below :—

PIANOFORTE Steel Wire, Glass-hardened, Plate 59, fig. 13.

$\mathfrak{H}$.	$\mathfrak{B}$.	μ.
0	85 (initial)	—
10·28	670	65
15·42	1,150	74
20·76	1,770	85
25·69	2,500	98
30·83	3,260	106
35·97	4,020	112
41·11	4,710	115
46·25	5,470	118
51·39	6,060	118
56·57	6,690	118
61·97	7,300	117
66·80	7,880	117
72·25	8,460	117
77·08	8,880	115
82·22	9,380	114
87·36	9,780	112
98·11	10,700	109

§ 31. *Effect of permanent strain on the magnetic susceptibility and retentiveness of Iron.*—In the experiments on wrought-iron hitherto cited the metal used was in every instance in the annealed state. If after being annealed the piece be subjected to a longitudinal stress sufficient to cause permanent set, and, the stress having been removed, the magnetic quality be then examined, a very remarkable change will be found to have taken place. Not only is the piece less susceptible, especially to weak magnetising forces, but the form of the curves connecting $\mathfrak{B}$ or $\mathfrak{I}$ with $\mathfrak{H}$ is greatly changed. The residual magnetism is far less than when the material was in the soft annealed state.

In the following experiment with a piece of soft iron wire 60 centims. long and 0·158 centim. in diameter, the direct magnetometric method was used. The wire was annealed and put through a cycle of magnetisation as in former examples. It was then demagnetised by reversals (§ 19) and hardened by loading it with a weight of 60 kilos., which produced a permanent extension of nearly 6 centims. The weight was removed and a cycle of magnetisation again gone through.

The observations have been reduced by calculating $\mathfrak{H}$ and $\mathfrak{I}$ (instead of $\mathfrak{H}$ and $\mathfrak{B}$ as in former examples; $\mathfrak{B}$ may be readily found from the relation $\mathfrak{B}=4\pi\mathfrak{I}+\mathfrak{H}$). Of course in calculating $\mathfrak{I}$ for the wire after stretching, allowance has been made for the change of dimensions of the piece. For the sake of easy comparison the numerical values of $\mathfrak{H}$ and $\mathfrak{I}$, and their ratio, κ, during the first application of the magnetising force, (1) before stretching, (2) after stretching, are given below. The force was in each case reversed, reapplied, removed, and reapplied as in former examples: but to avoid filling the page with tables of figures it will suffice to refer to fig. 14, Plate 59, where the whole experiment is graphically recorded.

SOFT Iron Wire, Annealed and Hardened by Stretching, Plate 59, fig. 14.

Before stretching.			After stretching.		
$\mathfrak{H}$.	$\mathfrak{I}$.	κ.	$\mathfrak{H}$.	$\mathfrak{I}$.	κ.
0	0	—	0	0	—
0·07	2		0·07	0	0
0·09	3		0·14	0	0
0·14	4	29	0·29	2	7
0·28	10	36	0·42	4	9
0·42	16	38	0·70	7	10
0·58	24	41	0·99	13	13
0·70	33	47	1·44	21	15
0·99	62	63	1·73	27	16
1·16	91	79	2·14	41	19
1·30	140	108	2·88	72	25
1·44	195	135	3·58	116	32
1·58	280	177	4·20	167	40
1·76	364	207	4·90	218	44
2·02	468	232	5·76	265	46
2·14	507	237	7·20	359	50
2·28	549	241	10·78	566	53
2·51	614	245	11·90	613	52
2·74	673	245	15·20	751	51
2·88	702	244	17·50	817	47
3·16	764	242	23·61	947	40
3·58	842	235	29·81	1017	34
4·20	926	220	35·71	1078	30
5·76	1020	177	41·90	1114	27
6·46	1050	163			
7·20	1070	149			
8·64	1110	129			
10·26	1130	110			
11·91	1150	97			
17·50	1190	68			
23·61	1195	51			
35·71	1230	34			
45·51	1230	27			

In fig. 14 the curves in full lines, giving the relation of $\mathfrak{I}$ to $\mathfrak{H}$ for the wire before stretching, will be readily distinguished from their resemblance to those which have already been described and shown in earlier figures. The lower and more sloping curves with more rounded outlines, drawn in broken lines, exhibit the behaviour of iron which has been hardened by stretching. The effect of stretching is that the subsequent magnetisation proceeds much less readily as the magnetising force is increased, but diminishes much more considerably as the force is withdrawn. The residual magnetism of the stretched wire is less than half that of the unstretched wire; but in the former it is much more stable, requiring nearly three times as much reverse magnetising force to remove it from the stretched as from the unstretched metal. The hysteresis in the stretched wire, during removal and reapplication of the magnetising force, is greater than in the case of annealed pieces.

Several other experiments of a similar kind have shown that the sloping curves of fig. 14 are thoroughly characteristic of strained iron. In fact the difference between them and the curves of magnetisation of an annealed specimen is so distinctly marked that it is easy, by applying this magnetising test, to distinguish a piece which has been strained beyond its limit of elasticity from a piece which has not been so strained.

A comparison of figs. 8 and 9 shows the same kind of difference between the curves for a steel wire when annealed (fig. 9), and when in its commercial hard-drawn state (fig. 8). The operation of wire-drawing gives a strain which differs from that to which the iron wire was subjected in the above experiment only in having lateral compression combined with the longitudinal extension.

§ 32. *Interpretation of* $\int \mathfrak{I} d\mathfrak{H}$ *in a Cycle of Magnetisation.*—The hysteresis which occurs in the relation of magnetisation in iron to magnetising force has been observed and commented on by Professor E. Warburg* in a paper with which I was not acquainted when the preliminary notice of my own work, referred to above (§ 1), was published. He has also anticipated me in pointing out the important physical value possessed by the *area* of curves representing the relation of $\mathfrak{B}$ to $\mathfrak{H}$ or $\mathfrak{I}$ to $\mathfrak{H}$ during cyclic changes of magnetisation.

The quantity $-\int \mathfrak{I} d\mathfrak{H}$ for such a cycle is, in absolute measure, the work spent in conducting the metal through its changes of magnetisation, per unit of volume. This may very readily be deduced from the general expression for the energy of the magnetic field given by Maxwell (El. and Mag. II., § 636) as

$$-\frac{1}{8\pi} S . \mathfrak{B}\mathfrak{H}.$$

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[*] Wied. Ann., xiii., p. 141. The rods experimented on by Warburg were, with one exception, scarcely long enough to satisfy the condition of approximate endlessness mentioned in § 7, as needful in experiments intended to answer the question raised in § 2. Reference should also be made in this connexion to recent papers by F. Auerbach, Wied. Ann., xiv. (1881), and C. Fromme, Wied. Ann., iv. (1878), and xiii. (1881). The papers of Auerbach and Fromme deal, under the name of "magnetische Nachwirkung," with the influence which previous magnetic state has on the actual magnetisation of iron and steel; but their mode of treating the subject differs considerably from that of Warburg. Auerbach distinguishes rightly between the "Nachwirkung" which is static, and that which (like the "elastische Nachwirkung" of strained india-rubber, or the "residual polarisation" of a dielectric solid) depends on time. It is the former of the two with which we are now concerned. Neither Fromme nor Auerbach used rods long enough to give an approach to uniform magnetisation.

In the present case $\mathfrak{B}$ and $\mathfrak{H}$ have the same direction. For any indefinitely small change of the field

$$dE = -\frac{1}{8\pi}(\mathfrak{H}d\mathfrak{B} + \mathfrak{B}d\mathfrak{H}).$$

When the change is cyclic, $\int\mathfrak{H}d\mathfrak{B}$ and $\int\mathfrak{B}d\mathfrak{H}$ are equal, and

$$\int dE = -\frac{1}{4\pi}\int\mathfrak{B}d\mathfrak{H}$$
$$= -\int\mathfrak{I}d\mathfrak{H}.$$

This is the energy expended in performing the cycle, per unit of volume of the substance magnetised, and it has a positive value for all actual cycles of magnetism in iron or steel in consequence of the hysteresis which exists in the relation of $\mathfrak{I}$ to $\mathfrak{H}$. The energy so expended is a thing quite apart from the further dissipation which occurs when, on account of the changes of magnetism taking place not indefinitely slowly, currents are induced in the substance of the magnet and in neighbouring conductors, and also from that which occurs in consequence of the thermomagnetic properties of the metal.* These latter sources of dissipation are functions of the time, and are absent when the magnetic change occurs indefinitely slowly: their effects do not appear in the cyclic curves of $\mathfrak{I}$ and $\mathfrak{H}$ when we leave a sufficient interval after each change of $\mathfrak{H}$ before taking a reading of $\mathfrak{I}$. The former, however, is independent of the rate of magnetisation, the hysteresis to which it is due being a static phenomenon; and if we accept the idea that the magnetic molecules of WEBER have their rotation opposed by a species of static friction, the dissipation of energy we are now considering is the work done in forcing them to rotate back and forth against this friction.

§ 33. *Numerical Values of* $\int\mathfrak{I}d\mathfrak{H}$ *for various Cycles.*—Measurements have been made of the values of $\int\mathfrak{B}d\mathfrak{H}$ in a number of the cycles of magnetisation in iron and steel which have been already described and figured. These, divided by 4π, give the corresponding values of $\int\mathfrak{I}d\mathfrak{H}$, or the energy expended, in c.g.s. units of work or ergs., in carrying one cubic centimetre of the material through the cycle in question.

	Ergs.
(1) For the large cycle of fig. 4 (§ 20), produced by the double reversal of magnetisation in a piece of very soft annealed iron wire from $\mathfrak{B}=13{,}450$, $\mathfrak{H}=17{\cdot}26$, to $\mathfrak{B}=-13{,}190$, $\mathfrak{H}=-17{\cdot}26$, and back, the value of $\int\mathfrak{I}d\mathfrak{H}$ is	9,300
(2) For the small loop of the same figure, produced by removing and restoring the magnetising force of $+17{\cdot}26$, the value of $\int\mathfrak{I}d\mathfrak{H}$ is . .	490
(3) For the large cycle of fig. 5 (§ 21), produced by the double reversal of magnetisation in a less soft annealed iron wire, from $\mathfrak{B}=15{,}710$, $\mathfrak{H}=22{\cdot}27$, to $\mathfrak{B}=-15{,}550$, $\mathfrak{H}=-23{\cdot}08$, and back, the value of $\int\mathfrak{I}d\mathfrak{H}$ is	16,300

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* Sir W. THOMSON, Phil. Mag., vol. v., 1878, pp. 24, 25.

Ergs.

(4) For the large cycle of fig. 6 (§ 22), produced by the double reversal of magnetisation in a piece of annealed iron wire similar to the above, but between higher limits of magnetising force, namely, from $\mathfrak{H}=90$ to -90, and back, the value of $\int \mathfrak{I} d\mathfrak{H}$ is . . . . * 16,700

(5) For the upper small loop of fig. 6, produced by removing and restoring the magnetising force of $+90$, the value of $\int \mathfrak{I} d\mathfrak{H}$ is . . 350

(6) For the cast-iron ring of § 24, fig. 7, the reversal of magnetisation from the greatest positive value there produced to an equal negative value, and back, gives as the value of $\int \mathfrak{I} d\mathfrak{H}$ 6,100

This, however, is obviously much less than the energy which would be expended in a double reversal of magnetisation in cast-iron, were the magnetisation as nearly complete as in the experiments on wrought iron and steel.

The above values of the energy dissipated in producing cyclic changes of magnetisation are greatly exceeded when the material is steel.

(7) For the hard-drawn steel wire of § 25, fig. 8, the double reversal of magnetisation between the extreme positive and negative limits ($\mathfrak{B}=$ about 14,600, $\mathfrak{H}=57\cdot5$), gives for the value of $\int \mathfrak{I} d\mathfrak{H}$. . . . 60,000

And the small loop, for the removal and restoration of $\mathfrak{H}$ in the same experiment, on the positive side, gives 6,300

(8) The corresponding quantities in the case of the same steel wire, annealed (§ 26, fig. 9), are :—

For the double reversal . . . . $\int \mathfrak{I} d\mathfrak{H} =$ 70,500

For the small loop . . . . $\int \mathfrak{I} d\mathfrak{H} =$ 1,800

(9) And in the case of the same steel wire, glass-hardened (§ 27, fig. 10):

For the double reversal . . . . $\int \mathfrak{I} d\mathfrak{H} =$ 76,000

For the small loop . . . . $\int \mathfrak{I} d\mathfrak{H} =$ 700

In this case, as in the case of cast-iron, the magnetisation was so far from "saturation" that a comparison of the above values of the integral with those of other experiments would be unfair.

In the three next examples the magnetising force was much greater than in the above, and the energy expended in going through the cycles was also greater. The measurements are those of §§ 28–30, figs. 11, 12, and 13, the material being pianoforte steel, and the magnetising force ranged from $+100$ to -100 nearly.

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[*] This value was inadvertently given as 1670 instead of 16,700, in a paper read before the British Association at Southport (Report for 1883, p. 404), and reprinted in the Phil. Mag. for November, 1883, p. 383.

(10) Pianoforte steel at normal temper (fig. 11):— Ergs.
For the double reversal . . . . $\int\mathfrak{I}d\mathfrak{H} = 116{,}000$
For the small loop . . . . $\int\mathfrak{I}d\mathfrak{H} = 3{,}500$
(11) The same, annealed (fig. 12):—
For the double reversal . . . . $\int\mathfrak{I}d\mathfrak{H} = 94{,}000$
For the small loop . . . . $\int\mathfrak{I}d\mathfrak{H} = 5{,}000$
(12) The same, glass-hardened (fig. 13):—
For the double reversal . . . . $\int\mathfrak{I}d\mathfrak{H} = 117{,}000$
For the small loop . . . . $\int\mathfrak{I}d\mathfrak{H} = 2{,}500$

The following measurements are taken from the experiment of § 31, fig. 14, in which an iron wire was subjected to a large cycle of magnetisation first in the annealed state, and then again after being stretched considerably beyond its limit of elasticity. The values of $\int\mathfrak{I}d\mathfrak{H}$ (for a double reversal of the greatest intensity of magnetisation) are:—

(13) For the wire before stretching . . . . . 10,000 ergs.
(14) For the same wire after stretching . . . . 16,400 „

The difference is considerable, but it is scarcely so striking as the other differences in magnetic behaviour which have been described in § 31 above.

Heating effect of cyclic changes of Magnetism.

§ 34. The energy expended in a cyclic process of magnetisation can take no other form than that of heat, diffused throughout the substance of the metal. Experiments have been made by Joule and others to determine by direct observation the heating effect of magnetisation in iron.* In most direct measurements of this quantity no distinction is made between the heating effect due to the induction of electric currents and that due to changes of magnetisation *per se*, and so excellent an authority as Professor Rowland, writing in 1881, has expressed himself as doubtful whether changes of magnetisation, considered apart from the currents they induce, give rise to any development of heat at all.† But no direct calorimetric measurements are needed to show that a cyclic process of magnetisation does give rise to a certain development of heat, even if it be performed so slowly as to make the dissipation of energy through

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* Joule, Phil. Mag., xxiii., 1843; Grove, Phil. Mag., xxxv., 1849; Villari, 'Nuovo Cimento,' 1870; Cazin, Ann. de Chim. et de Phys., 1875; Trowbridge, Proc. Amer. Acad. of Arts and Science, 1879. Other references are given by Warburg in the paper cited in the text.

† "I do not mean to here affirm that no heat can be due to the demagnetisation alone, but that we have at present no experimental proof of such direct transformation. All the experiments hitherto made have merely given us the heating due to induced currents."—Rowland, 'Scientific American, Supplement,' Nov. 5, 1881.

the induction of currents vanish. The positive value which the integral $-\int \mathfrak{J} d\mathfrak{H}$ always has is of itself conclusive evidence, and the magnitude of this quantity can be determined in absolute measure with an accuracy which would be unattainable in direct measurements of the heating effect which it represents. WARBURG, who has made determinations of the value of $-\int \mathfrak{J} d\mathfrak{H}$ in certain cases, has (along with L. HÖNIG, Wied. Ann., xx., 1883, p. 814) compared them with the results of direct calorimetric measurements, the difficulty of which is well illustrated by the poor agreement he finds between the two classes of observations.*

From the values given above for the integral $-\int \mathfrak{J} d\mathfrak{H}$ we may easily calculate the rise of temperature which a piece of iron or steel suffers when subjected to the cyclic changes of magnetism to which these values refer. Let us assume that the operation is conducted so slowly that $-\int \mathfrak{J} d\mathfrak{H}$ represents the whole work done, and that the heat produced is prevented from leaving the metal. Then, taking JOULE'S equivalent as 41,600,000 ergs. per gramme-degree (centigrade), the specific gravity of the metal as 7·7, and its specific heat as 0·11, we have—

Rise of temperature for every erg. expended per cubic centimetre

$$= \frac{1}{41{,}600{,}000 \times 7{\cdot}7 \times 0{\cdot}11}$$
$$= 2{\cdot}84 \times 10^{-8}$$

in degrees centigrade.

We have seen that the double reversal of a *strong* condition of magnetism in soft iron involves the expenditure of about 10,000 ergs. per cubic centim., and the consequent rise of temperature is therefore 0·000284° C. Nearly 4000 double reversals of a magnetic state approaching saturation would therefore be necessary to raise the temperature of a piece of soft iron by 1° C., if we could eliminate the action of induced currents in the metal.

The largest of the values given above (for the case of hard steel) is 117,000 ergs., which corresponds to a rise of temperature of 0·0033° C. per cycle. The rise in other instances of reversal of magnetism, and that caused by removal and reapplication of the magnetising force, may easily be found from the figures already given.

These developments of heat are so small as to make it apparent that the very considerable thermal effects which reversal of magnetism causes in the revolving cores of some dynamo-electric machines must be due almost wholly to the internal induction of currents, so far as they are not due directly to the current circulating in the coils

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* Another direct method of observing experimentally the dissipation of energy involved in cyclic changes of magnetism is to measure the "damping" of a swinging magnet by the induction of magnetism in an iron plate placed near it; but this also depends largely on the induction of currents. *Cf.* WARBURG, *loc. cit.*, also F. HIMSTEDT, Wied. Ann., xiv. (1881). HIMSTEDT has concluded from an experimental examination that the greatest part of this damping is due to what I term statical hysteresis, for the effects were found to be in great measure independent of the frequency of vibration of the needle.

of the armature. The experiments have a practical value in showing that cores which are so thoroughly laminated as to render the induction of currents within them unimportant, do not involve any serious loss of energy, and that the efficiency of a machine with a soft iron core or cores whose magnetism is periodically reversed need not, on that account, be materially less than that of a machine which has no such cores. The absence of iron from the armature has been claimed, on the score of efficiency, as an important advantage possessed by some types of machine, but unless the claim has some other basis it appears to me to be illusory. Magnetic reversal does involve some loss of energy, but if the cores are properly laminated so that the loss consists almost entirely of the quantity $-\int\mathfrak{J}d\mathfrak{H}$, it is so small as to be practically insignificant.

I shall show later that when a cycle of magnetisation is performed while the iron is kept in a state of mechanical vibration the value of $-\int\mathfrak{J}d\mathfrak{H}$ is much less than when the same cycle is performed with the metal in a state of rest, and indeed almost vanishes in soft iron. Hence, in a dynamo, where vibration occurs to a greater or less degree whenever the machine is running, the energy dissipated through changes of magnetisation is even less than these experiments on still metal might lead us to expect.

[Note added March 25, 1886.—The heating effect of cyclic changes of magnetism, especially of reversals, is important from a practical point of view not only in relation to dynamo-electric machines, but also to "secondary generators," or induction coils for the distribution of electrical energy by electromagnetic induction. In § 52 (below) mention will be found of occasional evidence which these experiments furnished that there is a true *time lag* in the magnetisation of iron: in other words, that there is a certain degree of viscous as well as static hysteresis in the relation of magnetism to magnetising force. Soft iron, especially in early stages of magnetisation, exhibits a sluggishness in assuming the magnetic state proper to the magnetising current—a sluggishness which does not appear explicable as an effect of the induction of currents within the substance of the iron, nor as due to the self-induction of the magnetising circuit. The result of this magnetic viscosity is to augment the value of $-\int\mathfrak{J}d\mathfrak{H}$ in any rapidly performed cycle, and consequently to increase the evolution of heat. It seems highly probable that in the comparatively slow reversals of magnetism which the core of a dynamo armature undergoes the additional dissipation of energy due to this cause is scarcely sensible. In the case of "secondary generators," however, where reversals of magnetism occur far more rapidly, there is nothing to show that this source of loss is not a sensible portion of the whole; but the high efficiency which this kind of apparatus has been proved capable of makes it at least clear that the magnetic viscosity of the core gives rise to no very serious loss of power.

In connexion with "secondary generators" and induction coils generally, the bearing of the first part of this paper should be noted, as showing the enormous advantage which a ring-shaped core, or core forming a complete magnetic circuit, possesses over a short bar core with ends. In an ordinary induction coil, so long as the current in the primary circuit is merely made and broken, a short core is necessary, since a ring core would lose but a small percentage of its magnetism at each break; but where reversal of the magnetising current takes place a core approximating to the condition of endlessness has an advantage, in respect of power, which fig. 3 makes obvious.]

§ 35. *Heating Effect of Lower Cycles.*—In most of the examples cited above (§ 33), in which the energy expended in cyclic changes of magnetisation has been determined, the magnetisation was strong, lying in the region of so-called saturation. It seemed desirable to measure the energy expended in carrying iron and steel through double reversals of weaker magnetisation, and accordingly the two following special experiments were made.

In the first a piece of annealed iron wire, 0·078 centim. in diameter and 29 centims. long, was tested by the direct magnetometric method. Starting initially with a state of no magnetisation, the magnetising force was slowly raised to 1·5, then slowly reversed to −1·5, and then slowly re-reversed to 1·5, numerous intermediate observations of $\mathfrak{I}$ and $\mathfrak{H}$ being made. The curves showing the relation of $\mathfrak{I}$ to $\mathfrak{H}$ in this process were afterwards plotted, and the area enclosed by them was measured. Then the magnetising force was further increased to 1·95, and another cycle performed by reversing and re-reversing it. Next it was raised to 2·56, and another cycle performed by double reversal. Next to 3·01, when another cycle was performed, and so on, until the last double reversal took place between the values + 75·2 and −75·2 for $\mathfrak{H}$. In this way 10 areas were measured, giving 10 values of the energy expended in reversing and re-reversing the magnetisation of the wire at different grades of intensity. The whole process is shown graphically in Plate 60, fig. 15, which is a very much reduced version of the diagram by help of which the areas were measured. It would take too much space to reproduce here all the readings taken in determining these curves, as no fewer than 338 corresponding values of the magnetisation and the magnetising field were measured. The following are the values of $\mathfrak{I}$, $\mathfrak{B}$, and $\mathfrak{H}$ at the successive points at which the magnetism was reversed in sign, and the corresponding values in ergs of $-\int \mathfrak{I} d\mathfrak{H}$ for the operation of double reversal. The last column gives the rise of temperature caused by one complete operation of double reversal of magnetism.

GRADED Cyclical Magnetisations of Soft Iron, Plate 60, fig. 15.

$\mathfrak{H}$.	$\mathfrak{B}$.	$\mathfrak{I}$.	$-\int \mathfrak{I} d\mathfrak{H}$.	Calculated rise of temperature.
			ergs.	deg. C.
1·50	1,974	157	410	0·000012
1·95	3,830	304	1,160	0·000033
2·56	5,950	473	2,190	0·000062
3·01	7,180	571	2,940	0·000083
3·76	8,790	699	3,990	0·000113
4·96	10,590	842	5,560	0·000158
6·62	11,480	913	6,160	0·000175
7·04	11,960	951	6,590	0·000187
26·5	13,700	1090	8,690	0·000247
75·2	15,560	1230	10,040	0·000285

If the relation of $\int \mathfrak{I} d\mathfrak{H}$ for double reversals of $\mathfrak{H}$ to the value of $\mathfrak{H}$ at which reversal takes place be plotted, it will be seen that the values of the integral appear to be approaching a limiting value, and the general form of the curve is not very different from that of a curve of $\mathfrak{I}$ and $\mathfrak{H}$ or $\mathfrak{B}$ and $\mathfrak{H}$. By plotting $\int \mathfrak{I} d\mathfrak{H}$ and $\mathfrak{I}$ it may, however, be shown that (except at the very beginning of the curve) the values of the integral, as the loops become larger, increase less rapidly than the values of $\mathfrak{I}$ between which reversal is made to take place.

§ 36. *Graded Cyclical Magnetisations of Steel.*—A similar experiment was made, also by the direct magnetometric method, with a piece of annealed pianoforte steel wire, 0·078 centim. in diameter, and 32 centims. in length. Seven double reversals were effected with magnetising forces graded up to 82 c.g.s. units. The resulting relations of $\mathfrak{I}$ and $\mathfrak{H}$ are shown in Plate 60, fig. 16. The loops formed by double reversal of $\mathfrak{H}$ were, especially with low values of $\mathfrak{H}$, far from closed (the same characteristic is noticeable in the curves for steel which have been already described), and for that reason the experiment scarcely allows comparative measurements of the successive areas to be made.

A noticeable feature in this diagram is the want of symmetry between the positive and negative sides. The positive values of $\mathfrak{I}$, coming as they did *first*, are greater than the negative values induced by an opposite and equal $\mathfrak{H}$. This characteristic is very conspicuous in the early parts of the operation, but disappears when the magnetisation becomes strong. The same feature is present in many other diagrams.

Ratio of Residual to Total Induced Magnetism in Iron and Steel.

§ 37. The large fraction of the whole magnetisation, which was found to survive the removal of magnetising force, especially in annealed iron, has already been alluded to. A series of special experiments were made to determine the amounts of residual magnetism in various specimens of iron and steel, when the induced magnetism was varied within widely different values.

These experiments were in most cases made by the direct magnetometric method. A straight piece of wire, long enough to approximate as nearly as was practicable to the condition of endlessness, was hung vertically behind the magnetometer in the manner described in § 18. The magnetising force was applied by means of a solenoid wound close to the wire, while the vertical component of the earth's field was neutralised by maintaining a constant current of the necessary strength in another solenoid wound on a tube which enclosed the first. The correct value of the current in the second solenoid was calculated with reference to the known value of the earth's force, and then verified experimentally by subjecting to the process of demagnetising by reversals (§ 19) a long piece of soft iron in the interior. This process gives complete demagnetisation when the piece subjected to it lies in a perfectly neutral field, but leaves a large residue when the field is not fairly balanced. It thus affords an accurate test of the exact neutralisation of the earth's vertical component.

In some instances the ballistic method was used, and then the rods experimented on were laid at right angles to the terrestrial magnetic field. A few experiments were also made by the ballistic method on rings. The results obtained from long rods by the two methods, and those from rings, were in substantial agreement.

Plate 60, fig. 17, shows an actual example of the direct magnetometric method of investigating the ratio of residual to temporary magnetisation. Starting from a state of no magnetism, a force $\mathfrak{H}$ was applied and removed, then again applied and carried to a somewhat higher value, again removed, again reapplied and carried to a somewhat higher value, again removed, and so on. The application and removal of the force was conducted very gradually, and in this particular example intermediate readings of the magnetism during the application and removal were taken, from which the curves in fig. 17 have been plotted. Generally, however, only the induced and the residual magnetism for each stage in the increase of $\mathfrak{H}$ were determined. The following table gives the successive values of the applied force $\mathfrak{H}$, the intensity of induced magnetism, and the residual magnetism which survived the removal of the force, and finally the ratio of the residual to the induced or temporary magnetism. This ratio increases with increased strengths of magnetisation, and in this case does not pass a maximum even at the highest value of $\mathfrak{H}$. The piece tested here was the soft annealed iron wire of fig. 5 (§ 21), 0·077 centim. in diameter, and 30·5 centims. or 400 diameters in length.

INDUCED and Residual Magnetism in Soft Iron, Plate 60, fig. 17.

$\mathfrak{H}$.	$\mathfrak{B}$ induced.	$\mathfrak{B}$ residual.	Ratio of $\mathfrak{B}$ residual to $\mathfrak{B}$ induced.
1·34	413	165	0·400
3·33	6,690	5,660	0·846
4·88	11,910	10,790	0·906
6·80	14,060	13,060	0·921
8·43	14,800	13,640	0·922
13·74	15,670	14,550	0·926
22·37	16,080	15,010	0·933

The ratio in this example is slightly greater than I ever observed it in other cases, though values of 0·87 and even 0·90 were not uncommon, and in one other example 0·92 was exceeded. It appears to be approaching a maximum. In many other instances, which will be cited below, this maximum was actually passed, by using sufficiently great magnetising forces.

§ 38. The series of double loops described in § 35, and shown in fig. 15, give another set of values of residual and temporary magnetism corresponding to a greater set of values of $\mathfrak{H}$, in an annealed iron wire (length $=29$ centims., diameter $=0·078$ centim.). The following are the values they yield of $\mathfrak{H}$, $\mathfrak{I}$ (induced) and the ratio of the residual to the induced value of $\mathfrak{I}$. The figures given refer to the positive side of the diagram.

INDUCED and Residual Magnetism in Soft Iron, Plate 60, fig. 15.

$\mathfrak{H}$.	$\mathfrak{I}$ induced.	Ratio of residual to induced.
1·50	157	0·651
1·95	304	0·781
2·56	473	0·824
3·01	571	0·842
3·76	699	0·837
4·96	842	0·836
6·62	913	0·825
7·04	951	0·805
26·5	1090	0·753
75·2	1230	0·683

Here a distinct maximum is passed, when the ratio is about 0·84, after which it falls very considerably as the magnetising force is strengthened.

§ 39. As an example of the ballistic method the case may be cited of a *ring* of iron wire 0·248 centim. in diameter, the diameter of the ring being 8·3 centims. The following values were obtained :—

$\mathfrak{H}$.	Ratio of residual to induced.
0·70	0·165
0·95	0·172
1·36	0·183
1·68	0·353
2·31	0·537
2·75	0·577
3·21	0·671
3·78	0·729
4·62	0·774
5·35	0·803
6·41	0·818
7·15	0·825
8·10	0·829

§ 40. The following series of determinations of total and residual magnetism was made by the direct magnetometric method with another annealed iron wire, 0·078 centim. in diameter, and 28·7 centims. in length. The results are given numerically in the table below. Here the maximum in the ratio was 0·906, but the subsequent falling off in its value was less than in the former case (§ 38), as the magnetising force was not raised so far.

$\mathfrak{H}$.	$\mathfrak{I}$ induced.	$\mathfrak{I}$ residual.	Ratio of residual to induced.
0·86	26	6	0·250
1·48	67	26	0·381
1·98	164	96	0·598
2·66	478	378	0·793
3·24	680	574	0·843
3·78	802	696	0·868
4·29	888	786	0·884
4·86	956	859	0·899
5·40	991	898	0·906
6·05	1036	918	0·890
6·81	1067	946	0·883
7·56	1100	966	0·881
9·29	1142	997	0·873
11·20	1166	1014	0·868
12·64	1180	1023	0·868
14·59	1196	1033	0·863
15·49	1203	1034	0·861
17·24	1212	1042	0.859

In another experiment the ratio of residual to induced magnetism in an annealed iron wire passed a maximum of 0·885 with $\mathfrak{H}=6$, and fell to 0·779 with $\mathfrak{H}=90$.

§ 41. Another very extensive series of observations of this ratio was made on the annealed iron wire of § 31, both before and again after it was hardened by stretching, as described in that paragraph. The influence of stretching in reducing the retentiveness as well as the magnetic susceptibility of the iron is very remarkable. Here, again, the method was the direct magnetometric one.

SOFT Iron Wire, Annealed and Hardened by Stretching, Plate 59, fig. 18.

Before stretching.				After stretching.			
$\mathfrak{H}$.	$\mathfrak{I}$ induced.	$\mathfrak{I}$ residual.	Ratio of residual to induced.	$\mathfrak{H}$.	$\mathfrak{I}$ induced.	$\mathfrak{I}$ residual.	Ratio of residual to induced.
0·42	16	3·9	0·24	0·42	3·6	0	0
0·58	24	6·6	0·27	0·99	13·1	2·9	0·22
0·70	33	9·9	0·30	1·44	21·1	6·5	0·31
0·99	62	24	0·40	1·73	26·9	11·8	0·38
1·16	91	46	0·50	2·14	41	15·3	0·38
1·30	140	85	0·61	2·88	72	32·7	0·46
1·44	195	133	0·68	3·58	116	61·7	0·53
1·58	280	209	0·74	4·20	167	98	0·59
1·76	364	283	0·78	4·90	218	132	0·61
2·02	468	380	0·81	5·76	265	167	0·63
2·14	507	418	0·82	7·20	359	225	0·625
2·28	549	455	0·83	10·78	566	327	0·58
2·51	614	513	0·84	11·90	613	348	0·57
2·74	673	568	0·85	15·20	751	381	0·51
2·88	702	598	0·85	17·50	817	399	0·49
3·16	764	650	0·85	23·61	947	414	0·44
3·58	842	711	0·85	29·81	1017	417	0·41
4·20	926	783	0·85	35·71	1078	419	0·39
5·02	984	832	0·84	41·90	1114	419	0·38
5·76	1020	848	0·83				
6·46	1050	864	0·82				
7·20	1070	877	0·82				
8·64	1110	897	0·81				
10·26	1130	910	0·80				
11·91	1150	913	0·80				
17·50	1190	929	0·79				
23·61	1195	929	0·78				
35·71	1230	933	0·76				
45·51	1230	933	0·76				

These results are also shown in Plate 59, fig. 18, where the full lines express in terms of $\mathfrak{H}$ the values of the induced and residual magnetism, and their ratio, *before* stretching, and the dotted lines show the corresponding quantities *after* stretching. The maximum which the ratio of residual to temporary magnetism passes is even more distinctly marked in the hardened than in the annealed condition. The curves of fig. 18 show very clearly that the residual magnetism approaches saturation much sooner than the induced magnetism does, as $\mathfrak{H}$ is increased. In the region of strong force, while the curves of induced magnetism are still rising considerably, the curves of residual magnetism are running very nearly parallel to the axis of $\mathfrak{H}$, and it is to this that the maximum in the ratio is due.

§ 42. Several experiments with other pieces of iron wire, first annealed and then hardened by stretching, have fully confirmed the results of the preceding paragraph. Plate 61, fig. 19, shows the relations of induced and residual magnetism, and of their ratio, to magnetising force in another piece of stretched iron wire, which was much more strongly magnetised than the wire in the former example. Here the ratio, after touching a maximum of 0·6 with $\mathfrak{H}=9$, fell to 0·32 when the magnetising force was raised to 92.

Other experiments on the same subject will be found below in §§ 109–112, where the influence of stress on the ratio of residual to induced magnetism is considered.

§ 43. My experiments were better adapted for finding the proportion of residual to temporary magnetism when the magnetisation was moderately strong than when it was very feeble. So far as can be judged from the curves and figures given above, the ratio is initially zero, that is to say, the earliest developments of magnetism, induced by a magnetising force rising from nothing, are entirely temporary, and disappear altogether when the force is withdrawn. But these experiments are inconclusive on this point, though they show distinctly that, in any case, very weak magnetisation is very slightly retained when the magnetising force is withdrawn, even by specimens of iron which will retain as much as 90 per cent. of stronger magnetisation. In steel the absence of retentiveness at low magnetising forces is still more marked. It is clear from this that if we are to ascribe retentiveness to a frictional sticking of the displaced molecules, we must complicate the hypothesis by believing that all or some of the molecules have a certain range of possible displacement within which this friction does not act, or that some of them are free from frictional retardation.

§ 44. *Ratio of Residual to Induced Magnetism in Steel.*—Corresponding observations were made on pianoforte steel, in the three states already mentioned, namely, in the normal commercial temper, annealed, and glass-hardened by sudden cooling. The wires tested were in each case 0·78 mm. diameter and about 30 centims. long.

For pianoforte steel in its normal temper we have the following values of induced and residual magnetism for a graded set of values of $\mathfrak{H}$. The ratio here passes a distinct maximum, and (in conformity with what has just been said) is, in the early part of the experiment, exceedingly small. The following are the actual magnetometer readings, corrected for the electromagnetic action of the magnetising solenoid. To reduce them to $\mathfrak{J}$ they have only to be multiplied by 6·06.

PIANOFORTE Steel Wire, Normal Temper, Plate 61, fig. 20.

$\mathfrak{I}$.	Magnetometer readings.		Ratio of residual to induced.
	Induced.	Residual.	
9·4	5·0	0·6	0·120
10·9	7·1	1·5	0·211
12·5	10·1	3·3	0·327
14·0	13·8	6·0	0·435
15·6	17·8	9·0	0·505
17·2	24·3	14·1	0·580
18·9	30·6	19·7	0·644
20·3	37·5	26·4	0·704
23·4	54·9	41·0	0·747
25·0	64·0	48·8	0·762
26·5	72·2	56·9	0·788
28·2	82·9	66·0	0·796
29·6	90·3	73·0	0·808
31·2	98·2	80·3	0·818
32·8	106·4	87·8	0·825
34·3	113·5	94·4	0·832
35·9	120·2	100·5	0·836
37·4	126·2	106·0	0·840
42·1	140·5	120·2	0·856
46·9	150·4	127·5	0·848
54·6	161·1	135·7	0·842
62·4	167·5	140·9	0·841
70·2	172·6	143·9	0·830
78·0	176·0	146·0	0·829
90·9	182·1	149·0	0·818

This experiment is shown graphically in fig. 20, where, as in the former cases, curves are given showing the relation of $\mathfrak{I}$ induced, $\mathfrak{I}$ residual, and their ratio, to $\mathfrak{H}$.

§ 45. Plate 61, fig. 21, gives in the same way the corresponding results for a piece of the same pianoforte steel wire, annealed. Here the ratio of residual to temporary magnetism touches a maximum of 0·805 with $\mathfrak{H}=23$, and falls to 0·71 with $\mathfrak{H}=97$.

§ 46. Finally, Plate 61, fig. 22, gives corresponding results for a piece of the same steel wire, glass-hardened. Here even the strongest magnetising force used did not saturate the wire sufficiently to allow the maximum in the ratio of residual to induced magnetism to be very distinctly passed. It appears, however, to have been reached, and its value was 0·69.

§ 47. *Summary of Conclusions as to the Ratio of Residual to Induced Magnetism in Iron and Steel.*—The results of this section of the experiments may be summarised as follows :—

(1) When the induced magnetisation of iron or steel is very weak it almost all disappears on the removal of the inducing field.

(2) When it is increased its capability of being retained also increases, very rapidly, and the ratio of residual to induced magnetism reaches a maximum.

(3) Roughly speaking, the intensity of magnetisation at which this maximum in the ratio appears is in the region of what WIEDEMANN has called the "Wendepunct," or point at which the ratio of $\mathfrak{I}$ to $\mathfrak{H}$ is a maximum (the point at which a tangent drawn through the origin meets the curve connecting $\mathfrak{I}$ and $\mathfrak{H}$.)

(4) The value of the maximum in the ratio of residual to induced magnetism is from 0·84 to 0·93 in annealed iron, nearly as great in annealed, hardened, or tempered steel, but much less in hard-drawn iron, where its value is more like 0·6.

(5) As the magnetisation is further increased the residual magnetism approaches saturation more rapidly than the induced, and consequently this ratio diminishes—slightly in soft iron, more in steel, and much more in hard-drawn iron.

By induced magnetism, in the above statements, is meant the value of $\mathfrak{I}$ which is induced when a magnetising force $\mathfrak{H}$ is applied gradually, and without mechanical disturbance, the piece under test being initially in a neutral state.

By residual magnetism is meant the value of $\mathfrak{I}$ which remains when, after being so applied, the magnetising, force is removed gradually, and without mechanical disturbance.

No magnetising force must act on the piece except the assumed force $\mathfrak{H}$, and consequently the conditions of the experiment are realised only when we use either endless magnets or magnets of such length that the field produced by the ends is sensibly inoperative throughout the greater part of their length. These conditions were fulfilled in the experiments which have been described, as far as it seems practically possible to fulfil them. Any actual sample of iron must be only imperfectly homogeneous, and consequently its magnetisation, in a uniform field, cannot be uniform, apart from the action of the ends. Probably no form of specimen can be found which is more nearly homogeneous, and at the same time uniform in section, than the form chosen in these experiments—a drawn wire with a diameter of about one millimetre.

At the same time, the magnetisation of these wires, though perhaps as nearly uniform in the central part of their length as is practicable in any experiment, could not have been absolutely uniform, nor were they quite free from self-demagnetising forces when the field due to the solenoid was removed. When we consider that under the necessarily imperfect conditions of the experiments as much as 90 and even 93 per cent. of the magnetisation of soft iron was retained on the removal of the solenoid's magnetising force, it seems far from unlikely that soft iron, if tested under ideally perfect conditions, would be found to have perfect retentiveness, that is to say, that none of the induced magnetisation would disappear on the gradual removal of all inducing force. This view does not admit of experimental verification or disproval: the experiments show that it is at least not untenable.

Effects of Vibration on Induced and Residual Magnetism in Iron.

§ 48. It is well known that mechanical disturbance makes iron take up more magnetism under an inducing force than it would take up if undisturbed, and that after the withdrawal of an inducing force mechanical disturbance reduces the residual magnetism. I was, however, unprepared to find these effects exhibit themselves so markedly as they did in certain specimens of annealed iron.

It has been already mentioned (§ 14) that after an annealed wire of soft iron had been magnetised, and the magnetising force removed, gentle tapping had the effect of almost completely removing the residual magnetism, great as that was. In fact, in dealing with very soft annealed iron, I generally found mechanical vibration—produced by gently beating against the table—to be the most convenient way of restoring the wire under test to its original neutral condition after magnetisation. The residual magnetism of soft iron is so sensitive to disturbance that in experiments made with the view of observing the retentiveness of the metal, the utmost care must be used to avoid accidental errors due to this cause. For example, a long rod of soft iron, to which a magnetising force had been applied and withdrawn, leaving about 90 per cent. of the induced magnetism as residual, was lying on a table in the laboratory, an induction coil round the rod being in circuit with a ballistic galvanometer. A person walking quickly along a neighbouring corridor caused by each footstep a distinct transient current in the galvanometer, due to the shaking out of successive parts of the rod's residual magnetism. The lightest tap or even rub by the fingers made the magnetism leave the rod with a rush; and during the application of magnetising force, the most apparently insignificant disturbance, occurring when the magnetisation was slight, would give rise to an enormous increase of magnetism, making it suddenly rise twenty-fold or more. For these reasons it is necessary, in experiments such as those which have been described in the preceding part of this paper, to guard very rigorously against accidental disturbance of the piece under test. Methods of measurement which involve the removal of the piece from the magnetising solenoid, or its movement in any way, before the magnetisation is determined, are quite out of the question. All the observations already described were made after I had become convinced of the necessity of preventing vibration, by supporting the rods and rings of iron on stone columns, detached from floors, and well out of the way of passers-by.

A number of separate experiments were carried out with the view of testing more particularly the effects of mechanical vibration on magnetic susceptibility and retentiveness. The results are given below.

§ 49. (March 17, 1882.) A long wire of soft iron, annealed, 0·158 centim. in diameter and 64 centims. or 400 diameters in length, was examined as follows by the ballistic method. The magnetising force was raised by steps to a certain value, at which the magnetisation was determined by summing up the ballistic throws due to the successive steps by which the magnetising force was increased. Up to this point there had been no disturbance. Then the wire was tapped, or rather beaten smartly against the table, the magnetising force being kept constant. The magnetisation was then, after tapping, measured by slipping off the induction coil. Next, the induction coil was replaced and the magnetising force again raised by steps, without disturbance, to a new value, the magnetisation being determined as before by summation of throws. Then the wire was again tapped, and the resulting magnetism again determined by slipping off the induction coil, and so on. A similar series of observations were made at various stages during the removal of the magnetising force. The results are given below, and graphically in Plate 61, fig. 23, where the full lines show the magnetic changes which occurred while the rod was left undisturbed, and the broken lines those which were produced by vibration. At the outset the wire had a magnetism ($\mathfrak{B}$) of 170, not extractable by tapping.

EFFECTS of Vibration on the Magnetism of Soft Iron, Plate 61, fig. 23.

	$\mathfrak{H}$.	$\mathfrak{B}$.	Change of $\mathfrak{B}$, caused by vibration under constant value of $\mathfrak{H}$.
On..	0	170	
	0·32	190	+6430
	0·32 (with vibration)	6,620	
	1·08	6,720	
	1·61	7,120	+4480
	1·61 (with vibration)	11,600	
	2·00	11,620	
	2·9	11,770	
	3·4	11,940	+1020
	3·4 (with vibration)	12,960	
	5·3	13,100	
	6·9	13,360	
	7·8	13,660	+ 580
	7·8 (with vibration)	14,240	
	11·2	14,330	
	13·3	14,420	
	16·5	14,560	+ 80
	16·5 (with vibration)	14,640	
Off..	11·6	14,600	
	8·9	14,530	
	7·2	14,470	−1430
	7·2 (with vibration)	13,040	
	5·2	12,970	
	3·1	12,910	
	1·77	12,790	−1350
	1·77 (with vibration)	11,440	
	1·16	11,320	
	0·77	11,290	
	0·33	11,260	−4380
	0·33 (with vibration)	6,880	
	0·23	6,880	
	0	6,880	−6560
	0 (with vibration)	320	

It is in the early part of the curve, the part where (under ordinary conditions) the magnetism has not yet begun to rise rapidly, that the effects of vibration are most remarkable. At later points the effects are still considerable, until we reach the region of approximate saturation, where, as might be expected, vibration produces little change.

The form should be noticed which the curve takes at each place when, after tapping, the process of increasing or decreasing $\mathfrak{H}$ is resumed without mechanical disturbance. Like the initial part of a normal curve of magnetisation these portions seem to start off tangent to the direction along which $\mathfrak{H}$ is measured, and it is only after a considerable change of $\mathfrak{H}$ has taken place that the magnetisation begins to change at all rapidly. This characteristic of the curves affords strong confirmation of the idea that retentiveness in soft iron is chiefly due to a resistance to the rotation of WEBER'S molecular magnets of a kind resembling the static friction of solid bodies.

§ 50. *Magnetisation of soft Iron with and without Vibration.*—In the next experiment the same piece of soft iron was again tested by the ballistic method. A pair of "on" and "off" curves were taken in the ordinary way without vibration, $\mathfrak{H}$ being raised by steps to 16·8 and reduced by steps to zero, while $\mathfrak{B}$ was measured by the summation of throws. Then, after the residual magnetism had been removed by tapping, a second pair of "on" and "off" curves were taken, in which the wire was vigorously tapped at each value of the magnetising force, and the resulting magnetism determined by slipping off the induction coil. The former pair are shown by full lines, the latter by broken lines (—·—·—·), in Plate 61, fig. 24. Finally, after the magnetisation was again raised to 8000, by tapping under a force of 0·6, the process of magnetising was continued *without vibration.* This last part of the operation is shown by the dotted line (················) in the figure.

It will be seen that the "on" and "off" curves with vibration are nearly coincident. The following readings refer to the "on" curve with vibration :—

$\mathfrak{H}$.	$\mathfrak{B}$.
0	240 (initial)
0·04	840
0·15	3,370
0·31	5,370
0·62	8,260
0·96	9,540
1·60	10,740
2·92	12,040
5·04	13,140
7·00	13,460
16·8	14,750

The enormously great magnetic susceptibility which soft iron exhibits when vibrated is worthy of special remark. In the above example a magnetising force of only 0·5 c.g.s. unit gave an induction ($\mathfrak{B}$) between 7000 and 8000, or say half the greatest magnetisation which it is practicable to produce by the strongest forces. The susceptibility is a maximum at or close to the beginning of the curve. There the value of μ or $\frac{\mathfrak{B}}{\mathfrak{H}}$ is not less than 20,000, and that of κ or $\frac{\mathfrak{I}}{\mathfrak{H}}$ is about 1600.

The slow beginning and subsequent rapid rise of magnetism which is characteristic of the process of magnetisation when performed without mechanical disturbance is entirely absent from the curve taken with vibration, but reappears when, at any point in the vibration curve, we stop vibrating and continue the change of $\mathfrak{H}$ with the metal in a state of rest. The great retentiveness shown by iron at rest has completely vanished. At the highest magnetising force reached the value of $\mathfrak{B}$ given with vibration is only a little greater than that given without vibration. This difference would probably diminish and ultimately disappear (as it should do by WEBER'S theory) if a very strong magnetising force were applied.

§ 51. *Effects of Vibration in hard-drawn Iron and in Steel.*—It is only very soft annealed iron that exhibits the influence of vibration in so marked a way. A piece of iron wire cut from the same bundle as the above, but *not annealed*, was subjected to the same set of changes of $\mathfrak{H}$, and behaved in the way shown in Plate 61, fig. 25. The processes were the same as in the example just quoted, and the figure explains itself. Here the retentiveness, though much reduced by vibration, is by no means destroyed, and a decided difference is found between the "on" and "off" curves with vibration. The former, too, shows a certain amount of inflexion near its starting-point. As the metal is here in the hard-drawn state its susceptibility and retentiveness (without vibration) are much less than when annealed (*cf.* § 31).

In steel, whether drawn, annealed, or tempered, the effects of vibration are similar in kind to those shown in fig. 25.

[Note added March 25, 1886.—A point of practical interest in this connexion is the circular magnetisation of iron telegraph lines by the signalling currents which traverse them, and the consequent retardation of signals. It was pointed out by FLEEMING JENKIN as early as 1865[*] that the experiments of GUILLEMIN showed a greater retardation of signals on land lines than could be accounted for, on THOMSON'S theory, by the electrostatic capacity and resistance of the lines; and it is now generally recognised that this additional retardation is due to the large amount of self-induction which a telegraphic circuit exerts on account of the circular magnetisation of the iron wire. The results of the present experiments may be applied to estimate the magnitude of the effects in question. In circular magnetisation we have, of course, the condition of endlessness very perfectly realised. Moreover, the almost incessant vibration to which telegraph lines are subject must have the effect of increasing the magnetic permeability to values not far short of those found in the above experiments, where the wires were tapped during the application of magnetising

[*] "On the Retardation of Electrical Signals on Land Lines," Phil. Mag., June, 1865.

force. We may, as an extreme case, applicable to a very soft iron wire strongly vibrated, take μ as 20,000 for any magnetising force of small intensity. Apply this to the case of a wire of ordinary size, say 4 mm. in diameter, and carrying a current of one-tenth of an ampère or 0·01 c.g.s. unit. At the circumference of the wire, where the magnetising force is greatest, its value is

$$\frac{2C}{r},$$

where C is the current and r the radius of the wire; and this value is independent of the distribution of the current throughout the section, so long as that is symmetrical with regard to the axis. The magnetising force is therefore equal to

$$\frac{2 \times 0{\cdot}01}{0{\cdot}2} = 0{\cdot}1 \text{ c.g.s. unit.}$$

With $\mu = 20{,}000$ this gives $\mathfrak{B} = 2000$ as the value of the magnetic induction in the outermost layer of the metal—a magnetisation nearly half as great as that of the armature of a good modern dynamo! Along with this circular magnetisation there is combined, in wires which run more or less north and south, a longitudinal magnetisation due to the earth's field, of an amount comparable with the above, so that the actual lines of induction form helices whose pitch, besides decreasing from the axis towards the circumference of the wire, shortens and lengthens with every fluctuation of current.

In a circuit consisting of two long parallel straight aërial conductors, each of radius r and permeability μ, separated by a distance b, MAXWELL's formula ('Electricity and Magnetism,' ii., § 685) gives for the self-induction per unit of length of the system the value

$$\frac{L}{l} = 2 \log \frac{b^2}{r^2} + \mu.$$

It is interesting to notice the comparative values which this quantity assumes when the conductors are, on the one hand, of non-magnetic metal, such as copper or phosphor-bronze, and on the other hand (again as an extreme case) of very soft annealed iron, kept in a state of vibration. To make a numerical comparison, we may take wires of 4 mm. diameter, separated by a distance of 20 centims. Then, for non-magnetic metal,

$$\frac{L}{l} = 2 \log \frac{20^2}{0{\cdot}2^2} + 1 = 19{\cdot}4.$$

And for soft iron, under vibration,

$$\frac{L}{l} = 2 \log \frac{20^2}{0{\cdot}2^2} + 20{,}000 = 20{,}018.$$

Hence, with the assumed data, the iron circuit has one thousand times the self-induction of the non-magnetic circuit. This throws light on the fact * that a notable

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* PREECE, British Association (Aberdeen), 1885.

increase in the speed of signalling becomes possible when copper is substituted for iron in a line whose electrostatic capacity and resistance are kept unaltered ; and that the much more rapid electric impulses which traverse a telephone circuit often become muffled beyond recognition when iron takes the place of phosphor-bronze.]

§ 52. *Time Lag in Magnetisation.*—A few miscellaneous points will now be noticed which presented themselves in the course of these experiments.

I have already said that the hysteresis which has been described as occurring in the relation of magnetisation to magnetising force is of a static character. Some evidence was, however, given that in addition to much static hysteresis there is a small amount of viscous lagging in the changes of magnetism which follow changes of magnetising force. I repeatedly observed that when the magnetising current was applied to long wires of soft iron, either gradually or with more or less suddenness, there was a distinct *creeping up* of the magnetometer deflection after the current had attained a steady value, as measured by the deflection of the galvanometer through which it passed. This action was sometimes so considerable as to oblige me to wait for some minutes before taking the magnetometer reading. The amount of this creeping is, however, very small compared with the static hysteresis. It occurs most conspicuously in the softest iron and at points near the beginning of the steep part of the magnetisation curve, and is much more marked in wires which are being magnetised for the first time after annealing, than in wires which have been previously magnetised, and demagnetised by the method of reversals. The action goes on too long to be ascribable to the self-induction of the circuit, and indeed occurs most noticeably when the changes of the current are effected gradually by means of the slide; and I do not think that it can be regarded otherwise than as evidence of true magnetic viscosity.

§ 53. *Comparative Effect of Sudden and Gradual Changes of Magnetising Force, and Resultant Effects of Cyclic Changes of Magnetising Force.*—It has long been well known from the experiments of VON WALTENHOFEN * and others that when a magnetising force is suddenly applied the induced magnetism is greater than when the force is gradually applied, and that when a magnetising force is suddenly withdrawn the residual magnetism is less than when the force is gradually withdrawn. My own experiments afford many confirmations of these facts, but they show that in very long uniformly magnetised rods the difference between the effects of sudden and the effects of gradual changes of magnetising force are not great. Thus, in the ballistic method, the difference in effect is not very material whether we produce a change of magnetising force in a series of numerous steps, or in a single step. The former process gives slightly, but very slightly, less total change of magnetism than the latter. And in the magnetometric method the magnetisation produced by raising the magnetising current continuously by means of a slide was found to differ but little from that given by a sudden establishment of the same current. The differences,

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* Pogg. Ann., 1863.

such as they are, are of the kind which VON WALTENHOFEN'S experiments would lead us to expect, but their magnitude is not such as to make results got by the ballistic method be materially different from results got by the magnetometric method.

In the following experiments the comparative effects of slow and sudden changes of magnetising force are involved, along with certain other phenomena which will now be described, and which are in part a mere confirmation of results already published by C. FROMME.*

§ 54. *Effects of Repeated Application and Removal of Magnetising Force.*—When a magnetising force is first applied, then removed, and then re-applied, whether suddenly or gradually, the resulting value of $\mathfrak{J}$ is somewhat higher than that reached by the first application. A third application gives a somewhat higher value, and so on, the effects apparently approaching an asympotic limit. This has been already shown by the experiments of FROMME. At each removal of the magnetising force the residual magnetism is also left somewhat greater than before. And this second action (the increase of the residual magnetism) exceeds the increase of the induced magnetism, with the result that the changes of magnetism between residual and induced *diminish in range* with successive removals and re-applications of the magnetising force.

The following observations were made by the ballistic method on a long piece of soft annealed iron wire. The readings are given without reduction to absolute measure; they relate to a point which falls early in the steep part of the curve of magnetisation.

Magnetising Current.	Throw of Ballistic Galvanometer.	Magnetism.
First made	203	203
broken	−53·6	149·4
made	+54·2	203·6
broken	−47·8	155·8
made	+48·7	204·5
broken	−45·7	158·8
made	+46·6	205·4
broken	−44·9	160·5
made	+46·1	206·6
broken	−44·0	162·6
made	+45·6	208·2
After many makes and breaks—		
broken	−42·6	
made	+43·1	
After many more makes and breaks—		
broken	−39·5	
made	+39·8	

Similar results were repeatedly obtained, both with freshly annealed wires and wires from which a previous strong magnetism had been shaken out by tapping. In curves

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* Pogg. Ann., Ergbd. vii., 1875, and Wied. Ann., iv., 1878.

showing the relation of $\mathfrak{B}$ or $\mathfrak{I}$ to $\mathfrak{H}$, the same thing exhibits itself in what may be called the over-closing of loops formed by removing and re-applying a given value of $\mathfrak{H}$. A good example of this is furnished by Plate 60, fig. 17, which shows how much more considerable the action now spoken of is at early than at late stages of the magnetisation.

The following experiment (also on annealed iron) shows that the same kind of action occurs when the current is slowly changed by the slide of § 18, and the magnetism is determined by a magnetometer.

Magnetising Current.	$\mathfrak{H}$.	Magnetometer Deflection.	$\mathfrak{I}$.
Gradually raised to 70	2·46	93	298
,, reduced to . . . 0	0	65	208
,, raised to 70	2·46	97	310
,, reduced to . . . 0	0	70	224
Then 100 sudden makes and breaks—			
Suddenly raised to 70	2·46	103	330
,, reduced to . . . 0	0	80	256

§ 55. *Effects of Repeated Reversal of Magnetising Force.*—When a magnetising force is applied and then repeatedly *reversed*, the changes of magnetism, instead of being strictly cyclic, form what may be termed unclosed loops. Instances of this are given by a number of the preceding figures, especially by Plate 60, fig. 16, which shows a series of these unclosed loops in the magnetisation of steel wire. The result is, as in the former case, that successive repetitions of the process give a gradually diminishing range of magnetic change. This action, like the last, occurs most conspicuously at points in the early part of the curve of magnetisation. The following observations were made specially to exhibit it, on a piece of annealed iron wire, 400 diameters long, by the magnetometric method.

Magnetising Current.		Magnetometer.	
0		0	
Gradually raised to . . .	+190	+146	Here there is gradual diminution of range. This part of the operation is shown in fig. 26.
,, reversed to . .	−190	−141	
,, ,, ,, . .	+190	+127	
,, ,, ,, . .	−190	−133	
,, ,, ,, . .	+190	+120	
,, ,, ,, . .	−190	−132	
Suddenly ,, ,, . .	+190	+124	Here there is an increase of range due to the suddenness of these reversals.
,, ,, ,, . .	−190	−136	
,, ,, ,, . .	+190	+123	
Fifty double reversals, then			
Suddenly reversed to . .	+190	+111	But after repeating the sudden reversals often enough the range becomes smaller than ever.
,, ,, ,, . .	−190	−127	
Then gradually ,, . .	+190	+108	And a *gradual* repetition of the cycle causes still a further reduction of range.
,, ,, ,, . .	−190	−126	

In the first part of the above operations, during the five gradual reversals of magnetising force, intermediate readings were taken, which enabled the curves shown in Plate 61, fig. 26, to be drawn. These show at a glance the manner in which the range of magnetic change diminishes. Sudden reversals, following on these, cause at first an increase of range, thus illustrating the comparative effects of gradual and sudden change of $\mathfrak{H}$, but on being repeated many times they reduce the range to a lower value than before.

§ 56. The same piece of wire was next subjected to a magnetising force about five times greater than the above, and was then demagnetised by reversals (§ 19). Experiments similar to the above were then made on it, when it was found that *the tendency to a diminution of range with repetition of a cyclic alteration of magnetising force had disappeared.* The diagram, Plate 61, fig. 27, shows the effect of applying, reversing, and reapplying the same magnetising force as in the former case, after the wire had been demagnetised by reversals. It shows that the changes of magnetism are now cyclic. The same result was given by other specimens, which when freshly annealed gave much diminution of range, but when demagnetised by reversals after the magnetising force had been raised to a high value, were found to have lost this property. In this respect, then, a wire demagnetised by reversals differs from the same wire in its primitive annealed state. It will be seen, too, by comparing figures 26 and 27, that the unsymmetrical susceptibility with respect to forces of opposite signs which exists in the annealed wire has given place to a very perfect symmetry after demagnetisation by reversals. Re-annealing the wire restored all the characteristics of the primitive state.

§ 57. Similar experiments with other specimens of wire, and at other points in the curve of magnetisation, gave results confirmatory, in every particular, of the above. When the magnetisation is strong, however, the differences produced by repetitions of the same process are insignificantly small: and it is in the early parts of the curve that we find the most striking effects. One or two other examples may be given very briefly.

SOFT iron wire, freshly annealed.

Magnetising Current.		Magnetometer.	
Gradually	100	+130	
,, ...	−100	−125	
,, ...	+100	+117	
,, ...	−100	−116	
Suddenly	+100	+109	Here the repetition of the reversal, although sudden, still causes diminution of range.
,, ...	−100	−114	
Fifty more double reversals, then—			
Suddenly	+100	+106	
,, ...	−100	−106	
Gradually	+100	+103	Here even after many sudden reversals a *gradual* reversal still causes a decided lessening of the range.
,, ...	−100	−104	

The following observations, made with another piece of annealed iron wire at a part of the curve very sensitive to the actions now spoken of, show well the reduction of range by reversals and then the rise of magnetism, induced and residual, produced by successive removals and re-applications of $\mathfrak{H}$. This last occurs in a very marked way after the range of magnetic change has been reduced by reversals of $\mathfrak{H}$. The two directions of the current will for brevity be distinguished as A and B. The changes were sudden, and the magnetism was determined by the direct magnetometric method. A want of symmetry is very noticeable here between the positive magnetisation due to the current A, which is first applied, and the subsequent negative magnetisation due to the equal and opposite current B.

	Magnetometer.	
Made A	+232	
,, B	−110	Diminution of range by reversals.
,, A	+180	
,, B	−101	
,, A	+172	
,, B	−100	
Twenty reversals, then—		
Made B	− 95	
,, A	+158	
Broke A	+150	Rise of magnetism (induced and residual) by successive removals and re-applications of $\mathfrak{H}$.
Made A	+200	
Broke A	+193	
Made A	+206	
Broke A	+201	
Twenty makes and breaks, then—		
Broke A	+205	
Made A	+209	
Then reversals again—		The diminution of range by reversals is again conspicuous.
Made B	−105	
,, A	+178	
Forty reversals, then—		
Made A	+163	
,, B	−105	
Broke and remade B	−136	
Ditto twenty times	−175	

Further experiments with the same wire showed, after the range was reduced by reversals, that a gradual removal and re-application of $\mathfrak{H}$ produced, though to a much less degree, the same effect as a sudden removal and re-application; the effect, namely, of increasing the magnetism.

§ 58. The magnetisation of steel exhibits, even more than that of iron, reduction of range with successive reversals of $\mathfrak{H}$, and want of symmetry between the values of $\mathfrak{I}$ induced by successively applied + and − values of $\mathfrak{H}$. Plate 61, fig. 28, shows the changes of magnetism which were undergone by an annealed steel wire when a magnetising force of 15 c.g.s. units was applied, removed, re-applied, reversed, and re-reversed twice. The want of symmetry between the positive and negative values of $\mathfrak{B}$ is very marked in this example: the steel has acquired a strong *set* towards the side of the first magnetisation.

Values of the Coefficients μ and κ.

§ 59. It follows from the form of the magnetisation curve, as has been pointed out by STOLETOW and others, that the coefficient of magnetic susceptibility κ, or $\frac{\mathfrak{I}}{\mathfrak{H}}$, and therefore also the permeability μ, or $\frac{\mathfrak{B}}{\mathfrak{H}}$, rises, as magnetisation proceeds, from a comparatively low value to a maximum, and then decreases continuously as $\mathfrak{H}$ is further raised, at least within such limits of $\mathfrak{H}$ as have ever been reached in actual experiments. The changes in κ and μ which occur during the process of normal magnetisation are best seen by ROWLAND'S method of plotting κ in terms of $\mathfrak{I}$, or μ in terms of $\mathfrak{B}$. Plate 62, fig. 29, gives two examples of this last mode of representation. One of the curves shown there refers to the annealed iron wire of § 21. In it the values of μ (which are given numerically in § 21) pass from 128 for a very low value of $\mathfrak{B}$ to a maximum of 2370 for $\mathfrak{H}=4{\cdot}5$, and down again to 705 for $\mathfrak{H}=22{\cdot}27$. In the other, which refers to the annealed iron wire of § 22, the early values of μ are not given, but the magnetisation has been pushed further by raising $\mathfrak{H}$ to 89·8, with the result of reducing μ to 183 and of giving the curve an upward inflection in its descending limb. That such an inflection occurs when the magnetisation is sufficiently increased has been already noticed and commented on by FROMME (Wied. Ann., xiii., p. 695). It makes the formula given by ROWLAND (Phil. Mag., xlviii., p. 339) fail to serve as an equation to the curve in this extreme portion.

§ 60. Plate 62, fig. 30, gives two curves showing, in the same manner, the relation of κ to $\mathfrak{I}$ in the iron wire of § 31, first in its annealed state, and again after it had been hardened by stretching. They are drawn to the same scale, and bring out well by contrast the great influence which permanent set has in reducing magnetic susceptibility. They are plotted from the observed values stated in § 31. Other curves of the same character may easily be drawn by aid of the data contained in previous paragraphs.

§ 61. Curves of this kind suggest several interesting theoretical questions regarding the results which we should find if we were able to extend indefinitely the range of observations, both towards vanishingly low and towards indefinitely high values of $\mathfrak{H}$.

It seems exceedingly probable, to judge from the portion of the curve of κ and $\mathfrak{I}$ which we can actually determine, that, if produced backwards, the curve would cut the axis of κ at a finite positive value of κ.* This, in other words, would imply that the susceptibility is not indefinitely small at the very beginning of magnetisation.

To reconcile such a result with the idea of a static frictional resistance to the

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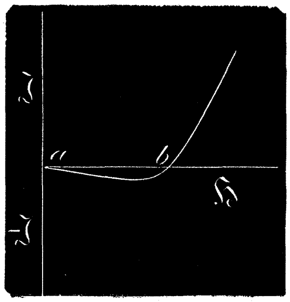
* ROWLAND has so produced his curves of μ and $\mathfrak{B}$ backwards, and has given values of μ corresponding to $\mathfrak{B}=0$. These, however, are merely hypothetical.

rotation of the magnetic molecules, we should be obliged to assume either that this friction was vanishingly small for some of the molecules, or that a certain range of motion was possible, to all or some of them, before the frictional resistance began to be felt. We have seen already that one of these assumptions would be required to account for the indefinitely small retentiveness which hard iron and steel seem to possess when exceedingly feebly magnetised.

But though the observed values of κ extend to very low values of $\mathfrak{I}$, between these and the zero of magnetisation there is room for a complete change in the form of the curve, and it would be rash to infer anything respecting the initial value of κ from the way in which the curve is trending at the finite magnetisations for which the lowest observed values of κ have been determined. The curves, indeed, give some slight evidence that a downward bend occurs near the origin, and, in any case, they afford no positive proof that κ is not initially zero, or even negative.

§ 62. If we could suppose that the frictional sticking of the molecules was at first complete, so that none of them budged from their primitive positions when a very low value of $\mathfrak{H}$ was applied, then we should expect to find the initial value of κ not zero, but a negative quantity. For in that case the cause producing paramagnetism (namely, the rotation of the molecules) would be entirely inoperative, and the application of $\mathfrak{H}$ might be expected to induce Ampèrian currents in the stationary molecules in the same manner as in other substances, with the result of making the magnetisation opposite in direction to the magnetising force, until the latter became large enough to overcome friction and begin turning the molecules. Thus, if none of the molecules could begin to turn until $\mathfrak{H}$ reached a certain finite value, we should expect to find iron *diamagnetic* with respect to lower values of $\mathfrak{H}$.

[Note added April 9, 1886.—It is possible that substances which are diamagnetic in fields of ordinary intensity are so, not because the molecular currents are absent or weak, but because the molecules are held so fast that their alignment (which would give paramagnetisation) is produced with difficulty. So long as they do not turn, or turn very slightly, diamagnetic induction is the resultant effect of the applied field.



If the resistance to the turning of the molecules resembles that found in iron, we should expect the diamagnetic induction which occurs when the field is moderate to give way to paramagnetic induction as the field is increased; also that a residual *paramagnetic* polarity would be left after a strong field has been applied and removed, even if the substance has remained diamagnetic during the action of the field. This view is supported by the experiments of LODGE ('Nature,' 1886, March 25, p. 484; also April 1, p. 512), which have shown residual paramagnetic polarity in copper and other diamagnetics after exposure to a strong field. According to this view the curve of $\mathfrak{I}$ and $\mathfrak{H}$ for substances in general is (in its earlier portion) of the form sketched. If the region *ab* extends so far as to include fields of considerable intensity, we call the substance diamagnetic. In iron, *b* comes so near the origin that we are always dealing with the further portion of the curve.]

§ 63. The extension of the curves of κ and $\mathfrak{I}$, or of μ and $\mathfrak{B}$, at the upper end is subject to a like uncertainty. As ROWLAND has remarked in the introduction to his second paper, we do not know whether $\mathfrak{I}$ or $\mathfrak{B}$, or either of them, attains a maximum; and MAXWELL* has given a reason (similar to the above) for supposing that, after passing a maximum, $\mathfrak{I}$ becomes negative. For, during the application of $\mathfrak{H}$, we may suppose that the induction of Ampèrian currents goes on in the molecules, giving a component of magnetisation opposite in sign to that which is produced by the turning of the molecules. The latter reaches a limit, the former increases indefinitely as $\mathfrak{H}$ is increased, and therefore ultimately exceeds the latter. Hence, iron may be diamagnetic under a very strong magnetising field.

In that case we should find that as $\mathfrak{H}$ is raised, $\mathfrak{I}$, after passing a positive maximum, decreases to zero, becomes negative, and then goes on always increasing negatively. The curve of κ and $\mathfrak{I}$, if produced by increasing $\mathfrak{H}$, would bend back, towards the origin, in its descending limb, pass through the origin and over to the negative side, κ never exceeding a small negative value as $\mathfrak{I}$ is increased negatively without limit.

§ 64. On the other hand, even if iron be ultimately diamagnetic under strong magnetising forces, we have no reason to expect that $\mathfrak{B}$ will pass a maximum. Since $\mathfrak{B}=4\pi\mathfrak{I}+\mathfrak{H}$, $\mathfrak{B}$ cannot pass a maximum with increase of $\mathfrak{H}$ unless $\frac{d\mathfrak{I}}{d\mathfrak{H}}$ attains a negative value greater than $\frac{1}{4\pi}$. This is a greater coefficient of magnetisation than any diamagnetic substance is known to possess, and, moreover, is too great to be consistent with WEBER'S theory of diamagnetism, according to which μ cannot be negative.

These considerations, if they serve no other useful purpose, show the futility of drawing conclusions as to the initial and ultimate values of the magnetic susceptibility of iron in indefinitely low and indefinitely high fields, from observations made, as all observations must be made, in fields of finite magnitude. For this reason it appears that an empirical formula, such as ROWLAND applies to the curves of μ and $\mathfrak{B}$, must be misleading when pushed beyond the range of actual experiment. If we do not know whether $\mathfrak{I}$ or $\mathfrak{B}$, or either of them, attains a maximum, it is a truism to say that there is no use in assigning numerical values to that maximum. The results obtained by extending a formula past the limits of experience

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* MAXWELL, 'Treatise on Electricity and Magnetism,' ii., §§ 843, 844.

have scarcely even a speculative interest if the basis of the formula is not an intelligible physical theory and its terms are not capable of physical interpretation.

§ 65. WEBER'S theory of the magnetisation of iron is incomplete, because it omits to take account of retentiveness. MAXWELL'S extension of it, which ascribes retentiveness to something resembling the permanent set of an overstrained solid fails, it appears to me, to explain the hysteresis which is found in all changes of magnetism brought about by changes of the magnetising force. The idea of a resistance to rotation of the molecules resembling the friction of solids, which has been suggested by several writers, suffices to explain hysteresis, and is supported by other phenomena which I have already noticed in passing. It accords remarkably well with the low values of κ which we find in the early part of the magnetisation curve; also with the effects which vibration has in increasing the susceptibility of iron to small forces and in destroying its retentiveness. In fact, for soft iron it requires little modification to make it cover the facts. In hard iron and steel, on the other hand, where the residual magnetism is of a far stabler character, retentiveness seems partly due to some such action as MAXWELL has suggested, though effects resembling those of friction are also present. Both notions—that of permanent set and that of frictional resistance—seem needful for anything like a full account of the phenomena, and if we are to attempt to form a mechanical conception of the process on such lines as these, we must assume: (1) An elastic tendency on the part of the molecules to recover their primitive position when displaced; (2) a static frictional resistance to their displacement and to their return, removable by vibration; (3) a limit for each, such that if the displacement of the molecule exceeds it, a permanent displacement, not removable by vibration, results; (4) probably a viscous resistance to the displacement and return of the molecules; (5) an unequal distribution amongst the molecules of the frictional resistance, such that in some of the molecules it is very (perhaps indefinitely) small. An alternative to this last supposition has already been mentioned, but it is in any case necessary to assume that the friction varies somewhat widely amongst the molecules, in order to prevent the susceptibility from changing discontinuously.

§ 66. A theory involving so many arbitrarily adjustable constants evidently admits of being brought without difficulty into general harmony with what we are able to observe of the process of magnetisation. Any examination of it by comparison with observed magnetic values could scarcely be conclusive. We might, however, by applying these ideas, construct, or at least imagine, a mechanical model in which the pieces would move in such a manner as to give results closely imitative of those observed when we magnetise and demagnetise iron. It must be admitted that any mechanical analogue of this kind is probably a very crude representation of molecular movements. One cannot suppose that if we could obtain an insight into the real nature of magnetic induction we should find anything at all resembling miniature drums turning under brake-straps; nevertheless, it is certain that the internal movements which are produced in iron by changes of magnetising force (and, as I shall show later, by other things besides magnetising force) do occur in ways which are not only extremely suggestive of the movement of solid bodies against frictional resistance, but are exactly analogous to such movements in some of their effects.

§ 67. In the molecular theory of magnetisation, as developed by WEBER and MAXWELL, the equation expressing the equilibrium of a magnetic molecule when deflected by a magnetising force, X, is

$$\mathrm{X} \sin \theta = \mathrm{D} \sin (\alpha - \theta),$$

where α is the original inclination of the molecule's axis to the line of action of X, θ is its inclination to the same line when deflected, and D is an assumed directive force which tends to keep the molecule in its primitive position.

We may introduce the idea of frictional resistance by writing this—

$$\mathrm{X} \sin \theta = \mathrm{D} \sin (\alpha - \theta) + \rho,$$

where ρ is a statical couple due to friction (to be reckoned per unit of magnetic moment of the molecule). In this form the equation will apply to cases where the deflection of the molecule is being or has just been increased by application of X. When the deflection is being diminished by reduction of X the term expressing the friction is to have its sign reversed.

The most easily affected molecules are those whose inclination to the axis of X is $\frac{\pi}{2}$. Hence, on beginning to apply magnetising force there is no deflection of any molecule until $\mathrm{X} = \rho$. Again, after magnetisation has been produced and the applied force X begins to be removed, no return of the molecules (that is to say, no loss of induced magnetism) occurs until the amount by which X is reduced exceeds 2ρ. The subsequent loss of magnetism as X is further reduced will depend on the relation of D to ρ.

Now, in soft iron the retentiveness is so nearly complete that D must be very small. We may examine the comparatively simple extreme case which we should find if D were equal to zero. Every molecule which is being turned by X must then satisfy the equation $\mathrm{X} \sin \theta = \rho$, and the action is always limited to those molecules for which $\sin \alpha > \sin \theta > \frac{\rho}{\mathrm{X}}$. For any given value of X, during the increment of X, the only molecules affected are those whose original inclinations lie between the values $\alpha_1 = \sin^{-1} \frac{\rho}{\mathrm{X}}$ and $\alpha_2 = \pi - \alpha_1$. Molecules lying outside these limits do not contribute to the resultant magnetisation of the piece.

Using the notation of MAXWELL ('Electricity and Magnetism,' vol. ii., § 443), in which m is the magnetic moment of a molecule, and n the number of molecules per unit of volume, we have

$$\mathfrak{I} = \int \frac{mn}{2} \cos \theta \sin \alpha \, d\alpha,$$

The limits are α_1 and α_2 as above, and $\cos\theta = \sqrt{1-\frac{\rho^2}{X^2}}$.

Hence,

$$\mathfrak{J} = mn\left(1-\frac{\rho^2}{X^2}\right)$$

This applies from $X=\rho$ to $X=\infty$. For values of X below ρ, $\mathfrak{J}=0$. mn is of course the magnetism of saturation. The following values of $\mathfrak{J}$ are calculated from this equation:—

X	$\mathfrak{J}$
ρ	0
$1{\cdot}25\ \rho$	$0{\cdot}360\ mn$
$1{\cdot}5\ \rho$	$0{\cdot}556\ mn$
$2\ \rho$	$0{\cdot}750\ mn$
$3\ \rho$	$0{\cdot}889\ mn$
$4\ \rho$	$0{\cdot}937\ mn$
$5\ \rho$	$0{\cdot}960\ mn$
$10\ \rho$	$0{\cdot}990\ mn$
$100\ \rho$	$0{\cdot}999\ mn$

By supposing that ρ is different for different molecules we can avoid the discontinuity which occurs at $X=\rho$; and the relation of $\mathfrak{J}$ to X deduced from the formula agrees fairly well with actual curves of $\mathfrak{J}$ and $\mathfrak{H}$ in specimens of soft annealed iron. Removal of X would of course give a straight line ($\mathfrak{J}$=constant), and application of X with sign reversed would give a rapid fall as soon as the value of the reversed force exceeded ρ. The results correspond so nearly with those found in soft annealed iron that it may be concluded that this extreme supposition (namely, that D is an insignificantly small quantity, and that retentiveness is due to ρ), although doubtless not very exact, does represent fairly well the behaviour of this material. No such simple theory will answer in dealing with hard iron or steel.

When we subject soft iron to vibration during the application and removal of the magnetising force, we cause ρ to vanish more or less completely, and the action which we observe agrees fairly well with the original theory of WEBER as developed by MAXWELL (*loc. cit.*, equations 5–8), and shows that D, though finite, is a very small quantity.

§ 68. *Numerical Values of the Maximum of μ and κ.*—The following table shows in a collected form, and in round numbers, the maximum values (deduced from these experiments) of μ and κ during initial magnetisation, and the value of $\mathfrak{H}$ at which they were found.

Specimen.	Maximum μ.	Maximum κ.	Value of $\mathfrak{H}$, giving maximum μ.
Moderately soft iron ring (of fig. 1, § 10)	1740	138	5·1
Soft iron ring (of fig. 2, § 11)	2300	183	2·6
Very soft annealed iron wire, 300 diameters long (fig. 3, § 15)	3500	279	2·6
Soft annealed iron wire, 400 diameters long (fig. 4, § 20)	2670	212	2·8
Annealed iron wire, 400 diameters long (fig. 5, § 21)	2370	189	4·5
Another similar piece (fig. 6, § 22)	2040	162	4·1
Steel wire, hard-drawn (fig. 8, § 25)	320	25	30
ditto annealed (fig. 9, § 26)	470	37	18
Pianoforte steel wire, normal temper (fig. 11, § 28)	273	22	39
ditto annealed (fig. 12, § 29)	295	23	28
ditto glass-hard (fig. 13, § 30)	118	9·3	55
Soft annealed iron wire (fig. 14, § 31)	3080	245	2·7
The same, after being hardened by stretching (fig. 14, § 31)	670	53	10·7
Annealed iron wire (fig. 17, § 37)	2470	196	4·5
Iron wire hardened by stretching (fig. 19, § 42)	450	36	17
Pianoforte steel (*bis*), normal temper (fig. 20, § 44)	258	20·5	38
ditto „ annealed (fig. 21, § 45)	415	33	20
ditto „ glass-hard (fig. 22, § 46)	126	10	55
Very soft annealed iron wire (fig. 24, § 50)	3500	279	2·1
The same wire, *with vibration during the application of magnetising force* (fig. 24, § 50)	20,000 (about)	1600 (about)	From 0 to 0·2

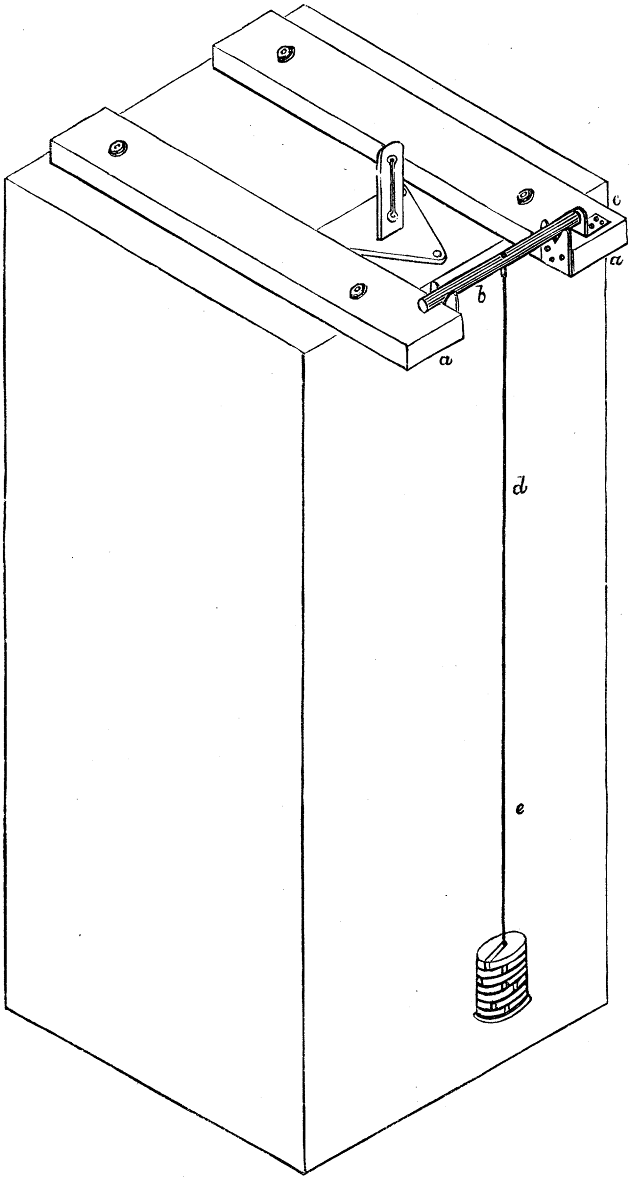
Effects of Stress on the Magnetic Qualities of Iron.

§ 69. *Method of Experiment.*—In all the experiments already described, in which an iron or steel wire formed the subject of test, the wire hung vertically under the action of no other stress than that due to its own weight. I shall now describe a series of experiments which were designed to study the variations of magnetic quality caused by pull applied in the direction of the wire's length, which was also the direction of magnetisation.

The wire to be tested was hung vertically with its upper end on a level with and due east of a mirror magnetometer. In my early experiments the desired longitudinal stress was applied by running mercury into a tank which hung from the lower end of the wire, but this plan was soon abandoned in favour of the much simpler one of stringing discs of lead on a copper wire which formed a continuation of the iron wire under test. Both in accuracy and convenience solid lead weights are preferable to a tank containing fluid, and I found it practicable to apply and remove the discs of lead with much less mechanical disturbance than was produced by running mercury into and out of a tank. This is an important point, for, as will be shown later, the influence of stress on magnetism is much affected by even the slightest vibration. Lead weights formed the load in all the experiments given below.

To enable the magnetometer zero to be tested from time to time it was desirable to have an easy means of removing the wire and replacing it in exactly the same position. The method, shown in the sketch, by which the wire was fixed, admitted of this.

The magnetometer stood on the top of a solid stone pier, which rose, clear of the floor, to a height of about 3 feet above the floor. To the top of it, and on the two sides of the magnetometer, two stout brackets, *a a*, were very rigidly fixed. To these a pair of brass plates with V notches cut in them were secured, which served to carry a cross piece, *b*, consisting of a cylinder of brass. This cylinder was prevented from moving in the direction of its own length by an end plate, *c*. The iron wire under examination, *d*, had brazed to it at its upper end a short piece of thicker brass wire which was rivetted into the middle of the cylinder, *b*. At the bottom of *d* a copper wire *e* was attached on which the lead weights were strung.



In some of the experiments the vertical component of the earth's magnetic field formed the magnetising force.* More usually magnetising force was applied by means of a solenoid which completely enclosed the wire: the strength of the current in the solenoid was measured by a mirror galvanometer, and was generally varied by means of the liquid slide described in § 18.

§ 70. The study of the effects of stress on the magnetic quality of a material consists in observing relations between the intensity of magnetisation and the two independent variables, magnetising field and stress. The effect of stress on residual magnetism is a special case, namely, the case of field $=0$. We may in all cases use one or other of two methods of inquiry: we may keep the field constant and vary the stress, or we may keep the stress constant and vary the field. The former method has been used by Sir WILLIAM THOMSON in a series of researches to which our present knowledge of this subject is principally due.† I have employed both methods, and where I have used the former plan I have observed not merely the resultant change of magnetism caused by the application or removal of a considerable amount of load, but the intermediate values of the magnetism when the total change of load is divided into a large number of steps.

The presence of *hysteresis* in all changes of magnetism which are caused either by change of field under constant load, or change of load under constant field, makes the phenomena to be examined exceedingly complex, and it is only when the results obtained by both methods are compared that we can obtain a moderately clear view of the subject.

Effects of Stress on Magnetism induced by the Earth's Vertical Force.

§ 71. An iron wire (of the same quality as that tested in § 21), 0·079 centim. in diameter and 29 centims. long, was hung in position and annealed there by the flame of a spirit lamp. Load was applied by stringing on discs weighing 1 kilo. each, and the magnetic changes (the inducing field being the earth's vertical force) were observed. The first application of load caused magnetic effects of a very complex character; and I shall reserve the account of them. But once the wire was somewhat stretched by a load which exceeded its limit of elasticity, the subsequent behaviour, when the load was removed and reapplied, was clearly marked and perfectly regular. Earlier and later experiments agree in showing that this behaviour, which will now be described, is thoroughly characteristic of *stretched* iron.

A total weight of 12·6 kilos. was applied, under which the wire stretched about 0·7 centim. Its section was then 0·48 sq. mm., and consequently every kilogramme of load caused a stress whose intensity was 2·08 kilos. per square millim. The load of 12·6 kilos. was applied and removed four times, during which the changes of magnetism gradually became nearly cyclic. Then, during the fifth application and

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* The value of this in Tokio, Japan, where the experiments were made, is 0·34 c.g.s. units.

† "Electrodynamic Qualities of Metals," Phil. Trans., 1856, 1875, 1879.

removal of the load the following observations were taken. The readings given are the deflections of the magnetometer needle from its zero: they represent on an arbitrary scale the intensity of magnetisation of the wire.

STRETCHED Iron Wire (March 2, 1882), Plate 62, fig. 31.

	Load in kilos. 1 kilo. = 2·08 kilos. per sq. mm.	Magnetometer reading.		Load in kilos.	Magnetometer reading.
Loading	0	252			
	1	259	Unloading	11·8	426
	2	273		11	428
	3	294		10	432
	4	323		9	435
	5	355		8	438
	6	389		7	440
	7	413		6	438
	8	430		5	431
	9	437		4	413
	10	438		3	385
	11·8	434		2	350
	12·6	429		1	304
				0	254

The same experiment is shown graphically in Plate 62, fig. 31, where the arrows distinguish between the *on* curve, or curve of loading, and the *off* curve, or curve of unloading. The whole action is very nearly cyclic, but there is a remarkable difference between the values of the magnetism corresponding to equal values of the stress during the *on* and *off* parts of the operation. The changes of magnetism lag behind the changes of stress.

This *hysteresis*, or lagging of the changes of magnetism behind the changes of stress, is of a purely static character. The magnetism does not change *after* a load has been applied, and while the load remains constant, except to a very insignificant extent. The successive steps in the process of loading and unloading may be performed at intervals of some hours even without affecting the form of the curves.

Examining now the *on* curve, we see that the effect of moderately loading a stretched wire is to increase its magnetism, under the conditions we have here, but that a maximum is passed, and the later stages of the loading diminish the magnetism. A similar maximum occurs on the *off* curve, but owing to the presence of hysteresis it occurs at a smaller value of the load than the maximum point in the *on* curve. Each maximum is, in fact, shifted by hysteresis to a later part of the operation than it would otherwise occupy, and if there were no hysteresis we should expect it to take a position between the two actual maximums, or at about 8 kilos. of total load. Another characteristic of the curves, obviously attributable to hysteresis, is the comparatively easy gradient at the beginning of the *on* curve and again at the beginning of the *off* curve.

§ 72. It is evident as a further result of hysteresis, that the relation of load to magnetism may, without any change either in the field or in the initial magnetic condition of the piece examined, have any one of the indefinite number of values defined by points lying between the *on* and *off* curves. For example, it is easy with a load of 4 kilos. to reach a value for the magnetometer reading anywhere between 323 and 413. The value of the magnetism depends not only on the actual load, but on the preceding states and changes of the load, and most particularly on the *immediately* preceding changes. To illustrate this more fully the following observations were made, on the same wire, after it had again been subjected to the cycle of loading 0—12·6—0. (That is, after a load of 12·6 kilos. had been again applied and removed.)

The successive changes of load passed through in this experiment were: 0—5—0—8—3—12·6—9—12·6—3—8—0, with intermediate steps, and the resulting magnetometer readings are given below :—

STRETCHED Iron Wire, Plate 62, fig. 32.

	Load.	Magnetometer.
Loading . . . . (*ab*)	0	245
	1	255
	2	270
	3	293
	4	322
	5	352
Unloading . . . (*bc*)	4	349
	3	338
	2	319
	1	290
	0	249
Loading . . . . (*cd*)	1	256
	2	271
	3	292
	4	321
	5	351
	6	384
	7	413
	8	430
Unloading . . . (*de*)	7	430
	6	427
	5	419
	4	405
	3	381
Loading . . . . (*ef*)	4	385
	5	393
	6	405
	7	419
	8	430
	9	438
	10	440
	11	436
	11·8	431
	12·6	425
Unloading . . . (*fg*)	12·6	425
	11·8	427
	11	430
	10	434
	9	437
Loading . . . . (*gh*)	10	434
	11	431
	11·8	428
	12·6	425
Unloading . . . (*hi*)	11·8	427
	11	430
	10	433
	9	436
	8	439
	7	441
	6	440
	5	433
	4	415
	3	388
Loading . . . . (*ij*)	4	392
	5	400
	6	410
	7	423
	8	432
Unloading . . . (*jk*)	7	432
	6	430
	5	424
	4	411
	3	390
	2	355
	1	306
	0	252
After half-an-hour	0	252

The same observations are plotted in Plate 62, fig. 32, which shows the loops formed in consequence of hysteresis by the *on* and *off* curves of subordinate cycles of stress, as well as by those of the main cycle. At each change from loading to unloading, or from unloading to loading, the new branch of the curve starts off in a direction tangent, or nearly so, to a line parallel to the axis along which stress is plotted. The subordinate cyclic operations 0—5—0, 8—3—8, 12·6—9—12·6, and 3—8—3 give rise to magnetic changes which are very nearly cyclic, and consequently produce scarcely any effect on the form of the main curves.

§ 73. *Influence of Vibration on the Effects of Stress.*—The indications of hysteresis in magnetic changes caused by stress disappear almost entirely when we submit the piece under test to mechanical vibration either during or after the changes of stress. The *on* and *off* curves then become nearly coincident. The range of magnetic change corresponding to any change of load is considerably increased. A maximum point is still found in the curve of magnetism and stress, but it is nearly the same for both (on and off) operations, and its position lies, as regards stress, between the positions which the maximums occupy when the operations of loading and unloading are performed without vibration. These results will be seen in the following observations, made on the same wire as the foregoing, and without any change of conditions, except that after each change of load the wire was vigorously tapped before the magnetometer reading was taken.

STRETCHED Iron Wire with Vibration, Plate 62, fig. 33.

	Load.	Magnetometer (after tapping).		Load.	Magnetometer (after tapping).
Loading . .	0	181	Unloading .	12·6	431
	1	237		11·8	434
	2	290		11	435
	3	348		10	440
	4	384		9	443
	5	415		8	445
	6	434		7	444
	7	447		6	429
	8	451·5		5	416
	9	451		4	389
	10	447		3	353
	11	442		2	306
	11·8	436·5		1	250
	12·6	431		0	192

The same observations are shown graphically in Plate 62, fig. 33. The on and off curves are not quite coincident: they still exhibit some hysteresis, but far less than when there is no vibration.

§ 74. An instructive method of studying the influence of vibration in destroying, or rather in greatly reducing hysteresis, is to partially load or unload in the ordinary way, and then, pausing at any stage in the process, to tap the wire. An immediate, often very large, change of magnetisation is the result. If the operation of loading or unloading be then resumed, hysteresis again becomes conspicuous.

The following observations form a good example of this method of procedure. They are plotted in Plate 62, fig. 34, where the full lines refer to the parts of the operation during which there was no vibration, and the lines drawn thus —·—·—· show the changes of magnetism produced by tapping. The wire was the same as that of §§ 71–73, and in the same conditions. A cycle of loading without tapping was gone through first. This brought the magnetometer reading, with no load, to 252. On tapping the wire this fell to 180. Loading was then begun, and carried on up to 5 kilos. without tapping. The wire was then tapped, when the magnetometer reading changed from 337 to 405. Loading was then resumed and carried on to 8 kilos. and the wire again tapped—and so on, as will be clear by inspection of the table :—

STRETCHED Iron Wire, with and without Vibration, Plate 62, fig. 34.

Load.	Magnetometer.		Load.	Magnetometer.	
0	252		12·6	423	after tapping
0	180	after tapping	11·8	425	without tapping
1	191	without tapping	11	429	
2	213		10	433	
3	251		9	437	
4	294		8	441	
5	337		8	450	after tapping
5	405	after tapping	7	452	without tapping
6	411	without tapping	6	451	
7	425		5	443	
8	440		5	425	after tapping
8	453	after tapping	4	419	without tapping
9	452	without tapping	3	400	
10	451		2	361	
11	447		1	322	
11·8	442		0	265	
12·6	436		0	181	after tapping

The agreement of points on the *on* and *off* curves, with tapping, is scarcely so good as in the experiment described in the last paragraph, probably because the vibration was scarcely vigorous enough. It is very interesting to notice how hysteresis reappears at each continuation of the loading or unloading, after the vibration has ceased, manifesting itself by the way in which each curve corresponding to such a continuation starts off tangent, or nearly tangent, to a line parallel to the axis of loads. Finally, in the descending limb of the off curve, the accumulated effect of hysteresis during the reduction of the load from 5 kilos. to 0 leaves the magnetometer reading higher by more than 80 divisions than the value which is reached by vibration at the zero of load. Nothing could illustrate better than this the manner in which hysteresis causes a diminution of the total range through which magnetism changes under varying stress, as compared with the range which we find when the variations of stress are accompanied by vibration.

§ 75. *Initial form of Curves of Magnetism and Load.*—It has been mentioned above that when a change from loading to unloading, or from unloading to loading, occurs, the new curve starts off tangent, or nearly tangent, to the direction in which loads are measured. In other words, the initial change of magnetism appears to be indefinitely small relatively to the change of stress whenever the operation is reversed from loading to unloading, or *vice versâ*. In the experiments which have been already cited the steps by which the load was increased or diminished were too large to allow the initial form of the curve to be very well defined, and as the matter seemed important as throwing light on the character of hysteresis, a set of observations were made in which the process of loading or unloading was conducted by very gradual stages at the places where the precise form of the curve was to be examined. A number of small weights were prepared, which could be added to or removed from the other weights hanging from the wire, wherever it was desired to have a number of closely consecutive points in the curve. Great care was taken in this experiment to avoid the slightest disturbance of any kind, since it was evident that the least trace of vibration would vitiate the results.

The wire, which was the same as before, and still hanging vertically in the earth's field, was loaded in the usual manner up to nearly 5 kilos.—a place where the *on* curve is very steep. The load was then removed, at first by very small parts, in order to determine as precisely as possible the initial form of the *off* curve. The following are the observations. The magnetometer scale was altered before this experiment, but the readings, although different from those taken in former experiments with the same wire, are still proportional to the total magnetisation. The wire had been slightly shaken before the loading began, and consequently the operation was not cyclic.

	Load.	Magnetometer.
Loading	0	89
	1	92·5
	2	100
	3	114
	4	132
	4·87	147
Unloading	4·78	147
	4·67	147
	4·43	146·5
	4	145·3
	3	141
	2	133
	1	118
	0	104

Here, then, it is clear that the *off* curve, beginning from a steep part of the *on* curve, starts off as nearly as can be observed tangent to the line of loads.

Similar observations were made at two other points, namely, at the upper end of the cycle (12·66 kilos.) and at a steep part of the *off* curve (2 kilos.). The results were as follows :—

Load.	Magnetometer.	Load.	Magnetometer.
11	188	7	191
11·78	186	6	192
12·22	184·7	5	189
12·66	183	4	183·6
12·57	183	3	172·6
12·46	183	2	155·7
12·22	183·1	2·09	155·7
11·78	184	2·20	155·8
11	185	2·44	156·1
10	186·7	2·87	157·8
9	188·5	&c.	&c.
8	190		

These three observations are plotted in Plate 62, fig. 35. The results render the conclusion very probable that, if we call $\mathfrak{J}$ the magnetism and P the stress, $\frac{d\mathfrak{J}}{d\mathrm{P}}$ is initially zero when the *change of stress* is altered in sign (from loading to unloading, or *vice versâ*), provided the operation be performed without mechanical disturbance.

§ 76. In all the above experiments on the effects of stress (§ 71 to § 75) the same wire was used—a wire which had been previously stretched beyond its limit of elasticity. To examine the effects of stress within the limit of elasticity on the magnetism of iron *in the annealed state*, and to trace the changes which occur during the passage from the annealed to the hardened state when the stress is carried up to and beyond the limit of elasticity, the following series of observations were made (March 3–4, 1882) with another piece of iron wire cut from the same bundle as the last. As before, the magnetising field 0·34 was in operation throughout.

The wire, after being hung in the vertical position behind the magnetometer which it was to occupy during the experiment, was annealed twice,—once with a small amount of weight hanging from it, which served to straighten the wire when it was heated to redness, and a second time after this weight had been removed. A weight of 1 kilo. was then applied and removed several times, during which the magnetisation rose somewhat. When the effects became cyclic the following readings were taken. As before, the magnetometer readings are proportional on an arbitrary scale to the total magnetism of the piece.

Load.	Magnetometer.
0	291
1	292
0	291
1	292

In this, the first, stage of loading, and with this small amount of stress, *on* gave *increase*, and *off* gave *decrease* of magnetism.

Then the loading was continued, with the result that the application of load up to 2 kilos. carried the magnetism through a maximum down to a value below the initial value, while unloading left it permanently raised :—

Load.	Magnetometer.
0	291
1	292
2	290
1	292
0	293

And another application and removal gave :—

Load.	Magnetometer.
0	293
1	292
2	290
1	291
0	293

Here, then, the effects having become cyclic, *on* gives *fall*, and *off* gives *rise* of magnetism, and the presence of hysteresis is quite distinct. This last action is repeated for all loads up to the limit of elasticity.

The cycles of loading and unloading were now progressively extended, by going 1 kilo. higher each time, but still keeping well within the limit of elasticity. The resulting magnetometer readings showed that, besides the alternate changes (*on* giving *fall*, and *off* giving *rise* of magnetism), there was a slight gradual *reduction* of the magnetism, due, doubtless, to some hardening of the metal; for the wire, although not visibly stretched, could scarcely fail to be permanently affected by the application of even these moderate amounts of load :—

SOFT Annealed Iron Wire in Earth's Vertical Field, Plate 62, fig. 36.

	Load.	Magnetometer.		Load.	Magnetometer.
Loading	0	293	Loading	0	287
	1	292		1	285¼
	2	290		2	282½
	3	286½		3	279
Unloading . . .	2	288⅓		4	275
	1	290½		5	270¼
	0	292¾		6	266
Loading	1	291		7	261
	2	289½	Unloading . . .	6	263
	3	286½		5	265½
	4	281		4	268½
Unloading . . .	3	283		3	272
	2	285⅓		2	275
	1	288		1	280
	0	291¾		0	286
Loading	1	290	Loading	1	284
	2	287½		2	281
	3	284		3	277½
	4	280		4	273
	5	274¼		5	269
Unloading . . .	4	277		6	265
	3	279		7	259½
	2	282		8	251½
	1	286	Unloading . . .	7	254
	0	290		6	256½
Loading	1	288		5	259⅓
	2	285½		4	262
	3	282		3	266
	4	278¼		2	270
	5	274		1	275½
	6	266		0	282
Unloading . . .	5	268			
	4	271			
	3	274			
	2	277			
	1	281			
	0	287			

These and the whole of the following observations detailed in this paragraph are shown graphically in Plate 62, fig. 36.

At this point a variation of the process was introduced. The wire was vibrated by tapping, which caused the reading to rise to 295, and the cycle of loads 0–8–0 was again applied with vibration before each reading of the magnetometer. The readings were as follows; they are also shown by the curves drawn thus —.—.—. in fig. 36.

	Load.	Magnetometer (after vibration.)
Loading	0	295
	1	296½
	2	289½
	3	281
	4	275½
	5	267
	6	258½
	7	249½
	8	236
Unloading	7	239
	6	243
	5	246½
	4	253
	1	265
	0	258½

It should be noticed here that a distinct maximum is passed, near the origin, before the fall of magnetism caused by increase of load occurs, and a similar maximum occurs on the *off* curve. The *off* curve lies quite clear of, and below the *on* curve, and the whole operation has resulted in a considerable reduction of magnetism. This is because the combined effect of load up to 8 kilos. and vibration has been to harden the wire somewhat, although no drawing out was noticed.

Next the cycle of loads 0-8-0 was again applied, this time without vibration, and it is interesting to notice that a passage through a maximum occurred with a load of about 2 kilos., a thing which would not have happened if the starting-point had not been altered by the previous tapping (see also fig. 36) :—

	Load.	Magnetometer.
Loading	0	258½
	1	261½
	2	263½
	3	262
	4	259
	5	254
	6	248½
	7	242½
	8	236
Unloading	7	237
	6	239½
	5	242
	4	245
	3	247
	2	250½
	1	254
	0	259

Then the application of load was repeated and carried a step further, to 9 kilos., and then (after removal) to 10 kilos. At that point the wire began for the first time to draw slightly, and a marked change occurred in the form of the curves of magnetism and stress. This will be apparent from the figures given below, as well as from the corresponding part of fig. 36.

Load.	Magnetometer.	Load.	Magnetometer.
0	259	0	255
1	257	1	253
2	255	2	251
3	252½	3	249
4	250	4	246½
5	247	5	243½
6	243½	6	240
7	239	7	236
8	234	8	232
9	229	9	227
8	231	10	218½
7	233	9	220½
6	236	8	223½
5	238½	7	225½
4	241	6	228½
3	244	5	231
2	248	4	234
1	251	3	237
0	255	2	239
		1	240½
		0	239½

The first of these two cycles agrees in character with the preceding ones; we are, in fact, still dealing with *annealed*, not *drawn* wire. But the cycle 0–10–0 begins the wire's passage from the annealed to the hard state, and the curves of magnetism exhibit a corresponding transition. The next stage in the loading, up to 11 kilos., caused a decided drawing out, and with it a complete transformation of the curves into the form which has been already described (§ 71) as characteristic of a stretched piece, and substantially the same form was retained with further successive additions of load. The load was progressively raised to nearly 15·8 kilos. Each step caused some additional stretching, and the act of stretching was accompanied each time by a fall of magnetism. The curves of fig. 36, showing the effects of successive cycles of loading, consequently became lower and lower as the operation was continued. The position of the maximum point, either on the *on* or the *off* curve, shifts out to the right—that is, it occurs with a higher value of the load, as the cycle of loads is extended. The broken lines in the figure refer to the changes which took place while the wire was in the act of stretching. At each stage in the stretching enough time was given for the wire to draw until the magnetometer reading became stationary. The stretching took place in several steps, and its final effect was to change the diameter of the wire from 0·79 to 0·76 mm. The highest load reached was 15·8 kilos., after which a second application of the cycle 0–15·8–0 was made. The observed readings are all given below, but to avoid confusion the final *off* curve after the last loading is not plotted in the figure.

SAME wire, becoming hardened by stretching, Plate 62, fig. 36.

Load.	Magneto-meter.	Load.	Magneto-meter.	Load.	Magneto-meter.	Load.	Magneto-meter.	Load.	Magneto-meter.
0	239½	0	137	0	119	0	112	0	104
1	239	1	140	1	121	1	115	1	106
2	239	2	147	2	127½	4·8	150	2	110½
3	239	3	158	3	137	5·8	164½	5·8	150½
4	238½	4	170½	4	151½	6·8	175½	6·8	161
5	237½	5	185	5	166	7·8	184	7·8	168½
6	235	6	196	6	180½	8·8	188	8·8	172½
7	231½	7	203½	7	191½	9·8	188	9·8	174
8	228	8	206	8	198	10·8	186½	10·8	173½
9	224½	9	206	9	198	11·8	184	11·8	172
10	219½	10	204½	10	197	12·8	181	12·8	169
11	201	11	201	11	195	13·8	178	13·8	166
10	202	11·8	198	11·8	192	14·8	174	14·8	163
9	203½	12·6	187	12·6	189	15·8	158½	15·3	161½
8	205	11·8	188	13·6	179	15·3	158¾	15·8	159½
7	207	11	189½	14	174½	14·8	159½	15·3	160
6	208	10	191	13·6	175	13·8	160½	14·8	160½
5	208	9	193	12·6	176½	12·8	162	13·8	162
4	204	8	195	11·8	177½	9	168½	12·8	163½
3	196½	7	196½	11	179	8	170	9	170
2	181	6	197½	10	181	7	171	8	171
1	163	5	196	9	183	6	169	7	172
0	137	4	191	8	184½	5	168	6	171
		3	179	7	186	4	162	5	168½
		2	165	6	186	3	153	4	163
		1	144½	5	183	2	140	3	152½
		0	119	4	176	1	123	2	140
				3	165½	0	104	1	124
				2	152			0	106
				1	134				
				0	112				

§ 77. The foregoing observations show with sufficient distinctness how a wire which has been stretched behaves, as regards magnetism induced by the earth's field, when successively loaded and unloaded. But the behaviour of the wire while still in the soft annealed state, that is before the stress has been allowed to pass the limit of elasticity, requires further elucidation. A number of other experiments have helped in the analysis of the somewhat complex phenomena which are produced by loading and unloading an iron wire while in the annealed state.

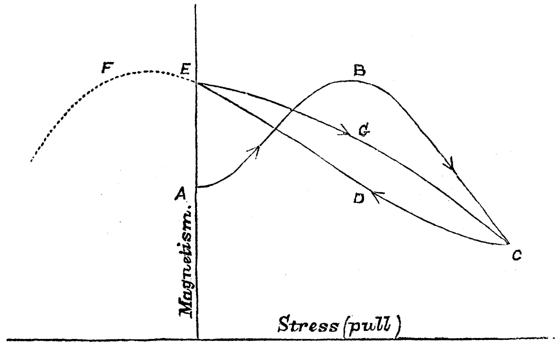
When the wire is annealed in the magnetising field, the amount of magnetisation which it possesses when it has become cool is an uncertain quantity, and depends much on the rapidity of the cooling, being greater for slow than for fast cooling. As soon as loading begins the magnetism rises rapidly, especially when it has chanced (perhaps from the cooling having happened somewhat quickly) to have been initially low. In fact, the annealed wire is, magnetically, in an extremely unstable state, and responds with great sensitiveness to the first application of load, every step of which (up to a certain limit) causes a rise of magnetism. This rise is not lost on the withdrawal of the stress; the application and removal of load leaves the wire permanently more magnetic. Indeed, the first application and removal of a small load produces an augmentation of magnetism not unlike that which would be produced by tapping. This continues during several successive loadings, and it is only when the load has been applied and removed many times that the changes of magnetism become sensibly cyclic.

§ 78. Taking now the particular case of the annealed iron wire already dealt with, the character of the effect of load when it becomes cyclic depends on the limit to which loading has been carried. If the total load has been only 1 kilo. (corresponding to an intensity of stress of say 2·1 kilos. per sq. mm.), then its application gives increase and its removal gives decrease of magnetism, when the cyclic state is reached.

But if the load has been 3 kilos., or anything greater (still not exceeding the limit of elasticity), the effect when it becomes cyclic is that *on* gives fall and *off* gives rise of magnetism. This corresponds to the effects shown in fig. 36 for the cycles of load from 0–3–0 up to 0–9–0.

Now it would seem, from the fact that when 1 kilo. only is applied and removed *on* gives rise and *off* gives fall of magnetism, that there is essentially a maximum of magnetisation in this annealed wire corresponding to a very small load, and this view is confirmed by curves taken with vibration, for in these a maximum appears at or near 1 kilo. of load. But this maximum does not appear in cyclic curves given by applying and removing higher values of the load. Its absence is, however, satisfactorily explainable as one of the effects of hysteresis. In fact, it still exists, but hysteresis shifts it to the left of the origin, so that successive applications and removals of pull lie on one side of it. If we could trace the effects of push as well as pull, the presence of this maximum would be immediately detected. The diagram below will make my meaning more clear.

Starting from A (a point reached, say, by tapping the annealed wire, when the load is zero), and beginning to load, we find the magnetism change in the manner represented by A B C. Then on removal of the load, the curve is C D E, E being the value of magnetism corresponding to no load. The *off* curve is still rising at this point, and my explanation of this rise, and of the apparent absence of a maximum point, is that the maximum point is shifted to the left by hysteresis so much as to fall outside the field of the operations, namely, to the left of the origin. If we could extend the experiment by applying *push* we should expect to find a continuation of the curve such as that sketched in a dotted line, with a maximum point between E and F. But of course on reapplying the load from zero the curve followed is E G C, and any succeeding repetition of the cycle gives a loop like E G C D. On the other hand, if at the beginning we had loaded with 1 kilo. only, *on* would have given rise and *off* would have given fall of magnetism in any repetition of the cycle, since in that case the whole action would have been confined to the ascending branch of the curve, namely, between A and B.



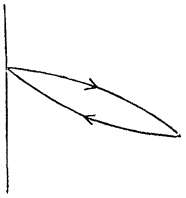
In this way, then, hysteresis causes the maximum which would otherwise occur at or near a load of 1 kilo. to disappear when cyclic loading is continued with greater loads. It also raises the value of the magnetism found with load $=0$ to a value which may fairly be called abnormal (as at E in the figure above), so that subsequent tapping under no load causes a fall of magnetism.

§ 79. If this explanation of the apparently anomalous actions which occur during the loading and unloading of an annealed wire be correct, the principal difference between the curves before and after stretching is in the position of the maximum point. Before stretching the maximum lies so near the origin as to disappear in the manner just described when the loading is repeated more than once. As the range of load is increased it shifts towards the right, and becomes prominently visible when and after the limit of elasticity is reached. It continues to shift towards the right as the wire is drawn out by successive increments in the range of stress. We thus find on careful examination not only continuity between the results obtained before and after stretching, but an essential similarity.

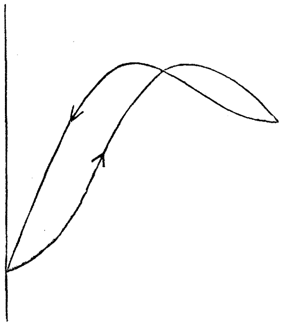
At the same time, experiments both with this and with other specimens of iron have shown that, when loaded and unloaded until the action becomes cyclic, a piece which has not been strained beyond its elastic limit since annealing behaves after the type of curve No. 1 below, and one which has received permanent set behaves after the type of No. 2.

This difference of behaviour appears, in fact, to form a criterion by which we may without fail distinguish a strained from an annealed piece.

No. 1.



No. 2.



§ 80. To illustrate the subject of § 78 the following experiments may be cited. The wire of § 76, now 0·76 mm. in diameter, was re-annealed in the vertical position three times over. When it had become cool, the magnetometer reading was found to be 159 (from the zero of magnetism). Then 1 kilo. was applied, and the reading jumped up to 220. Removal of the weight brought it to 218. The same weight was reapplied and removed several times, each time with the result that *on* gave rise and *off* gave fall of magnetism, while the successive operations gradually increased the magnetism. Then the range of loading was extended. Thus :—

SOFT Annealed Iron Wire in Earth's Vertical Field, Plate 62, fig. 37.

Load.	Magnetometer.	
0 (initial)	159	
1 (first application)	220	Sudden rise of magnetism by first application of load.
0	218	
1	222	
0	220¼	Here *on* gives rise and *off* gives fall of magnetism.
1	224	
0	222	
1	225½	
2	247	Note here the sudden rise of magnetism caused by this extension in the range of loading.
1	248	
0	248	
1	248	
2	248	
3	249	
2	251	
1	253	
0	254	
1	253¾	Here we have distinctly reached the condition in which *off* gives rise and *on* gives fall of magnetism.
2	252	
3	250½	
2	252	
1	254	
0	255½	
1	255	
2	253	
3	251½	
3	246	After tapping the wire.
2	248½	Without tapping.
2	275	After tapping.
1	277	Without tapping.
1	299	After tapping.
0	301	Without tapping This shows well how hysteresis leaves the value of the magnetism for zero load abnormally raised.
0	281	After tapping
*		
1	284	Without tapping.
1	295	After tapping.
2	293	Without tapping.
2	289	After tapping.
3	286	Without tapping.
3	272	After tapping.
2	275	Without tapping.
2	286	After tapping.
1	289	Without tapping.
1	304	After tapping.
0	306	Without tapping.
0	279	After tapping.
3	281	Without tapping.
0	287	” ”
3	281	” ”
0	287	” ”

---

* Up to this point these observations are plotted in Plate 62, fig. 37. The remaining operations have been omitted from the figure to avoid confusing it. The dotted curve showing a maximum at about ¾ kilo. is a curve drawn through the points reached by tapping. The tapping was not violent, in order that the soft wire should be kept from injury, and for that reason the succeeding *on* and final *off* curves with tapping do not nearly coincide.

§ 81. To examine further the state of instability which is reached by annealing in the magnetic field, the wire was again annealed with no load on it; then on cooling—

Load.	Magnetometer.
0	295
1	311
0	309
1 kilo. on and off five times, then	
0	309
1	311

In this instance, perhaps because of slower cooling, the initial magnetism was much nearer its full normal value than in the last case, and a cyclic state was reached after a single application of the load.

The wire was again annealed, by being heated to redness and allowed to cool very slowly. The reading was then 316, and the application of 1 kilo. raised it only to 317.

Finally the wire was again heated to redness, and cooled as quickly as possible (in air). The magnetometer reading was then exceptionally low, and the first application of load caused a great rise.

Then, as before, successive applications and removals of

1 kilo. gave *on* +, *off* −
2 kilos. gave doubtful, scarcely visible effects,
3 kilos. gave *on* −, *off* +
4 kilos. gave *on* −, *off* +.

§ 82. The same wire was again annealed, and the cycle 0—4 kilos. —0 was performed several times, until the changes of magnetism became cyclic. Observations were then taken during the cycle 0—4—0 which gave the curves *a b c d a* of Plate 62, fig. 38. Then the same cycle was repeated, with vibration of the wire before each observation, and the curves *e f g h e* were obtained. Finally, starting at *e*, the cycle 0—4—0 was again performed twice without vibration. This gave the curves *e i j k l m n o*.

These observations confirm what has been already said as to the existence of a maximum point near the zero of stress in the effects of stress on the magnetism of annealed iron. The maximum, which had been obliterated by hysteresis in the cycle *a b c d a* appears again in the vibration curves, and is again seen to disappear when the process of loading and unloading is resumed, without vibration. The curves *i j k l m n o* show the manner of this disappearance very plainly, and support the conclusions of § 78. A few more repetitions of the cycle would restore completely a simple loop like *a b c d*, but at a higher part of the diagram, since during vibration the magnetisation has been permanently raised.

§ 83. *Effects of Stress on Residual Magnetism.*—Throughout the foregoing examination of the effects of stress the vertical component of the earth's magnetic force (about 0·34 c.g.s. units) was acting longitudinally on the pieces tested. The effects observed were therefore due in part to the influence of stress on the inductive susceptibility of the iron, and in part to the influence of stress on the existing magnetisation. In order to study the effects of stress on residual magnetism a solenoid was placed round the wire throughout its whole length, and a constant current from a gravity DANIELL's cell was kept up in the solenoid, its strength being adjusted, by the introduction of suitable resistance, to a value which produced within the solenoid a field just equal and opposite to the vertical component of the terrestrial field.[*] This left the wire free from longitudinal magnetising force, and the changes of its residual magnetism, caused by applying and removing loads, were then observed by the magnetometer as before.

A piece of the same iron wire as before (0·79 mm. in diameter and 33 centims. long) was annealed and placed within the solenoid, in which, however, there was at first no current. The wire was loaded several times, up to 8 kilos., with the earth's vertical force acting, and was also tapped, with the result that at the zero of load the magnetometer showed a deflection of 450. Then a current neutralising the field was made in the solenoid, and the magnetism of the wire fell to 440.[†] This residue diminished quickly when loads were again applied, as follows, but several repetitions of the cycle 0—8—0 brought about a cyclic state of matters, of the kind which has been already described as produced in annealed iron by loading and unloading under the action of a constant field.

§ 84. Then the range of load was extended. Each addition caused a fall of residual magnetism; especially when (at 12 kilos.) the limit of elasticity was reached and the wire began to draw. At 13 kilos., when the wire had drawn about 5 per cent., it was unloaded and the cycle 0—13—0 repeated. The curve already described as characteristic of the effects of stress on a stretched wire in an inducing field reappears here as equally characteristic of the effects of stress on a stretched wire when there is no inducing field, and when the magnetism which is varied is wholly residual. The observations are as follows: they are also shown in Plate 63, fig. 39.

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[*] This adjustment was tested at intervals by removing from the solenoid the wire under examination and substituting in its place a wire of specially soft annealed iron. If tapping reduced the magnetism of this wire to zero the adjustment of the current was correct.

[†] This proportion of residual to induced magnetism, namely, $\frac{440}{450}$ or 97·8 per cent., is higher than in any of the experiments described in the earlier part of this paper. It must, however, be noted that in this case the wire was tapped while the inducing field was acting, and then the field was removed without tapping.

EFFECTS of Stress on Residual Magnetism (April 11, 1882), Plate 63, fig. 39.

Load.	Magnetometer.*	Load.	Magnetometer.
0	450	9	263
	in earth's field	10	253
0	440	11	244
	residual	12	233
1	436	13	192
2	429	Wire drew about 5 per cent.	
3	410		
4	387	12½	192
5	360	12	193
6	335	11	193½
7	317	10	194
8	302	9	195
7	303	8	195
6	306	7	193½
5	310	6	189½
4	314	5	181
3	318	4	166
2	324	3	146
1	329	2	127
0	330	1	103
Cycle 0—8—0 made six times; then		0	81
		1	84
0	300	2	89
1	298	3	96
2	296	4	107
3	292	5	121
4	288	6	134
5	284	7	147
6	279	8	155
7	274½	9	160
8	270	10	162
7	272	11	162
6	275	12	161
5	278	13	159½
4	282	12	160
3	285	11	161
2	290	10	162
1	294	9	163
0	300	8	163¼
1	297½	7	163
2	295	6	161
3	291½	5	157
4	287	4	148
5	282	3	135
6	279	2	118
7	274	1	98
8	269	0	78

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* In this series, one division of the magnetometer scale corresponds to $\mathfrak{I}=0{\cdot}77$ in c.g.s. units befo stretching, and $\mathfrak{I}=0{\cdot}8$ after stretching. The initial value of $\mathfrak{I}$ was, therefore, 346. The final reading of 78 divisions gives $\mathfrak{I}=62$. Each kilo. of load corresponds to a stress of 2·08 kilos. per sq. mm. before stretching, and 2·18 after stretching.

§ 85. The above operations are shown on the left hand side in fig. 39. The curves on the right hand side of the same figure refer to the following subsequent operations, for which the observed numbers need not be given. After several repetitions of the cycle 0—13—0, still acting only on the residual magnetism, the curves *ab* were taken. Then the earth's force was allowed to act (that is to say, the neutralising current in the solenoid was broken) and the cycle 0—13—0 was applied twice. The results of the second application are shown in the curves *cd*. Comparing *cd* with *ab* we see that in the inducing field the range of change of magnetism caused by change of stress is greater than when we are dealing with residual magnetism only, and the former changes occur of course at a higher position in the diagram than the latter; but the general character of the curves is substantially the same in both cases. At the end of the cycle *cd* the magnetometer reading was 140, and on tapping the wire this fell to 105, showing how, as a result of hysteresis, the value of the magnetism for zero of load was abnormally raised.

Next the neutralising current was again applied, and the wire was tapped (load = 0). The residual magnetism fell to 53. Loading and unloading then gave the curves *ef*, which, in consequence of hysteresis, left the residual magnetism at the end *greater* by 18 scale divisions than it was (after tapping) at the beginning. Tapping again reduced it to very nearly its former value.

Finally, another cycle of loads was applied with vibration before each reading, but the wire was broken at its bottom end by too violent tapping during the removal of load. The curves for this operation are *g* and *h*. Although during the operation the residual magnetism was suffering a great additional reduction by the combined effects of tapping and stress, a maximum of residual magnetism occurred in *g* at an intermediate value of the stress, as the curves taken without tapping lead us to expect.

It appears, then, that the effects of stress on the magnetism of iron, annealed or drawn, are substantially the same, whether the magnetism be induced or residual. In the latter case the effects are complicated by the gradual working out of the residual magnetism by successive loadings, just as in the former case they are complicated by the gradual working in of the induced magnetism, but when any operation is repeated often enough to give nearly cyclic effects this complication is eliminated, and we then find the effects of stress to be substantially identical in the two cases.

§ 86. *Effects of Stress on Magnetism induced by Fields of Various Strengths* (April 13, 1882).—The iron wire of the last paragraph (diameter 0·77 mm.), which had been broken close to one end, was again used, the length being now 30·5 centims. It was not re-annealed, but tested in the drawn condition to which it had been brought by the operations already described. By gradually increasing the current in the solenoid surrounding the wire, the magnetising field was brought to the value 3·33 c.g.s. units; during this time there was no load on the wire. Loading was then begun, and every addition caused at first an enormous increase of magnetism. Thus :—

	Load.	Magnetometer reading.	$\mathfrak{I}$.
	kilos.		
Field raised to 3·33	0	173	144
Began loading	1	246	205
	2	341	292
	3	467	389
	4	586	409
	5	678	565
	&c.		

At about 10 kilos. a maximum was passed. The load was increased up to 13 kilos., then removed, and the cycle 0—13—0 was repeated three times, with the result that the changes of magnetism became nearly cyclic. At the third repetition the following readings were taken :—

Load.	Magnetometer.	$\mathfrak{I}$.	Load.	Magnetometer.	$\mathfrak{I}$.
0	682	569	13	837	697
1	690	575	12	844	703
2	711½	593	11	853	711
3	751	626	10	863	719
4	794	662	9	873	727
5	836	697	8	883	736
6	869	724	7	892	744
7	890½	742	6	901	751
8	899	749	5	905	754
9	897	747	4	900	750
10	887	739	3	879	732
11	873	727	2	835	696
12	856	713	1	774	645
13	837	697	0	690	575

This cycle is shown in Plate 63, fig. 40, where it is marked IV. The dotted line running up through the diagram shows the effect of the loads which were first applied after the magnetising force had been brought to the value 3·33, at which it was kept during the remainder of the operation. Here, as in previous experiments to which attention was drawn in § 81, the first process of loading finds the magnetism in an exceedingly sensitive state, ready to respond by leaps and bounds to the successive increments of stress. The conditions, however, are different in the two cases. The wire of § 81 had previously been annealed in the magnetic field. In the present instance the field had been gradually increased to a constant value before the load was applied. We know already that the induced magnetism so reached is exceedingly sensitive to mechanical vibration (*cf.* § 49), and we now find a similar sensitiveness to application of stress, although unaccompanied by any vibration.

When the action is repeated often enough to bring the changes of magnetism to a nearly cyclic state, the form of curve with which we are already familiar is given, with a maximum at 8½ kilos. on the *on* branch, and at 5 kilos. on the *off* branch.

The wire being now without load, the magnetising current was next reduced to the value just necessary to neutralise the earth's vertical force—in other words, the field was reduced to zero—and the effects of stress on the residual magnetism were examined. The effect of the first application of the cycle 0—13—0 on the residual magnetism is shown in the curves marked IV*a.* in fig. 40. We see in it the resultant of two actions, viz., the usual cyclic effect and, superposed on it, a rapid reduction due to magnetic instability. On applying a load of 1 kilo. the fall of residual magnetism (resembling the fall which would be caused by tapping, or the rise which either tapping or loading would cause in the magnetism induced by an increasing field) more than counterbalances the small augmentation of magnetism which (in the cyclic condition) would be the effect of this load, and a curious dip at the beginning of the *on* curve is the result. Then the augmentation by stress becomes relatively more influential, until the maximum is passed, after which the two actions combine to give a rapid fall of magnetism as the load is further increased.

The cycle 0—13—0 was again applied twice, and, when the residual magnetism had settled into a nearly steady state, the cycle shown at IV*b.* in fig. 40 was taken.

The same wire was then subjected to several other magnetising fields, and cycles of loading performed in them. The results, in each case after the cycle 0—13—0 had been repeated several times, are shown in fig. 40, where the curves are numbered as follows :—

No. of curve.	Magnetising field (c.g.s. units).
I.	0·34
II.	1·66
III.	2·17
IV.	3·33
V.	5·17

The curves show the values of $\mathfrak{J}$ and the total load, which can be reduced to kilos. per sq. mm. by multiplying it by 2·18.

§ 87. (May 5, 1882.) In the above experiment the range of magnetising force was somewhat limited, and the following experiments were made to test the effects of stress on higher values of magnetisation.

In this and subsequent experiments the method of demagnetising by reversals of a gradually diminishing current was used to bring the wire to an initially neutral state between successive operations. The method has been described in § 19.

The wire was the same piece of iron as before (§§ 83–86), but before these experiments it was re-annealed and then stretched by a load of 15½ kilos. which stretched it from 30·4 to 31·8 centims. The effects of loading up to 15 kilos. were examined in three fields, viz., 0·34, 2·49, and 18·65 c.g.s. units. The operations in each case were as follows :—

(1) Demagnetisation by reversals with no load on.
(2) Application of the magnetising field with no load on.
(3) Loading and unloading (0—15—0) until a nearly cyclic state was reached.

The effects of the first loading in each field are shown by dotted lines in Plate 63, fig. 41, and the cyclic effects reached after several repetitions of the loading are shown by full lines in the same figure.

Thus the point *a* (fig. 41) is the magnetism reached by applying a force 0·34 without load. The curves *ab*, *bc* are the effects of loading to 15 kilos. and unloading in this field. The full lines just above them are the still very imperfectly cyclic effects given by a repetition of the cycle 0—15—0 in the same field.

Similarly the points *d* and *e* are the values of magnetism reached by applying the fields 2·49 and 18·65 respectively; the dotted lines starting from these points show the effects of the first loading in those fields; and the full lines above them the cyclic effects after two or three loadings.

The curves *fgh* show the effects of stress on the residual magnetism left after the removal of the field 2·49. The wire has been more stretched here than in the experiment of § 86, and, probably for that reason, appears to be somewhat more persistently retentive in the present case.

It is interesting to notice here how the susceptibility to the action of stress, as regards both the first general augmentation and the subsequent cyclic changes of magnetism, is greater in the intermediate field of 2·49, than in either the strong or the weak field. In the weak field the magnetic effects are still not cyclic after many repetitions of the cycle of load. In the high field, on the other hand, a sensibly cyclic state is reached after a single loading.

The maximum point occurs at a lower load as the strength of the field is raised.

The general effect of putting *on* 15 kilos. is positive for the low field, more strongly positive for the intermediate field, but negative for the high field. This reversal of the effects of *on* and *off*, by increase of the magnetising field, was noticed first by VILLARI, and afterwards independently by THOMSON, who has since named it the "Villari reversal."*

§ 88. To study more particularly the forms of the curves of magnetism and load during the passage of the effects of stress through a maximum, and the reversal of these effects in high fields, the following series of observations was made, in which a variety of magnetising fields ranging up to 34 c.g.s. units were employed.

(May 8th, 1882.) The same wire was further stretched under a load of 18·5 kilos., suffering an additional elongation of 3 per cent. After demagnetisation by reversals, with no load on, a magnetising field was gradually applied, and then kept constant during the changes of load. Loads were applied, up to 18·5 kilos., removed, reapplied, and so on. The load was then removed, the wire demagnetised by

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* VILLARI: Pogg. Ann., CXXVI., 1868; THOMSON: Phil. Trans., 1879, p. 55.

reversals, a new field applied, and the operation of loading and unloading resumed. The results are shown in Plate 63, fig. 42, and it would lengthen this paper needlessly to give all the recorded numerical values. The effects of the first loading after the application of each magnetising field are shown in dotted lines: the effects of a cycle 0—18·5—0 after several repetitions had brought about a nearly cyclic state of matters, are shown in full lines. The wire was demagnetised by reversals before the application of each new magnetising field. The fields were:—

Curve (in fig. 42).	Magnetising field (in c.g.s. units.)
I.	0·34
II.	2·49
III.	5·75
IV.	8·6
V.	11·6
VI.	20·1
VII.	34·0

One kilo. of total load corresponds to 2·3 kilos. per sq. mm.

This series of curves shows well the gradual transformation which occurs as we increase the intensity of magnetisation, before applying cyclic changes of stress. As the region of magnetic saturation is reached the maximum points on the *on* and *off* branches occur with lower and lower values of the load, and the descending branch of the *on* curve (to the right of the maximum) becomes a more and more important part of the whole. In the highest curve (No. VII.) it constitutes, in fact, nearly the whole action, and the *off* maximum point comes very near the zero of load, with the result that the subsequent *on* curve starts from so high a point as to exhibit no maximum at all. To the last, however, the initial curve of loading, after the field has been applied, shows a maximum point, though that also comes nearer the zero of load as the wire becomes more strongly magnetised. The resultant effect of the first loading passes a maximum, but remains positive in all but the highest curve; and after the cyclic state is established the general effect of *on* load is positive up to curve V., and has its maximum in curve II. The following table shows some of the results deducible by a comparison of the curves:—

EFFECTS of Stress on a Stretched Iron Wire in Constant Magnetic Fields, Plate 63, fig. 42.

No. of curve.	(1) Magnetising field, $\mathfrak{H}$.	(2) Initial $\mathfrak{I}$ value of before loading.	(3) This was changed by first application of 18·5 kilos. to	Difference (3) − (2) showing effect of first *on*.	Cyclic effects of 18·5 kilos.			Value of load in kilos giving *maximum* of $\mathfrak{I}$.		
					Value of $\mathfrak{I}$ for *off*.	Value of $\mathfrak{I}$ for *on*.	Difference *on* − *off*.	During first loading.	During cyclic *on*.	During cyclic *off*.
I.	0·34	7	75	68	110	183	73	14	10·7	7·0
II.	2·49	50	590	540	463	614	151	11	9·8	6·1
III.	5·75	195	740	545	642	748	106	10	9·6	5·9
IV.	8·6	372	842	470	776	845	69	9·8	9·5	5·7
V.	11·6	527	907	380	870	910	40	9·6	9·4	5·4
VI.	20·1	875	1045	170	1115	1045	− 70	8·4	8·2	4·7
VII.	34·0	1185	1140	− 45	1270	1141	−129	4·8	No maximum.	1

The numbers in column (2) are the values of $\mathfrak{I}$ reached by applying each magnetic field to the previously demagnetised wire. They correspond to the starting points of the dotted lines in fig. 42.

By comparing figs. 40, 41, and 42, as well as from fig. 36, we see that the position of the maximum point is shifted out (that is, towards higher values of the load) the more the wire is stretched. It is not, however, shifted out so far as to prevent the descending (right hand) limb of the curve from becoming relatively larger as the range of loading is extended by successive stretchings. This limb is most prominent in fig. 42, where the range of loading is extended to 18·5 kilos., although the maximum points occur there at higher loads than in figs. 40 and 41, in which the amount of stretching was less.

§ 89. *Similar Experiment with a Soft Annealed Wire.* — Only one more experiment of the same class as the foregoing need be referred to in detail. It was made (May 26, 1882) on another specimen of the same kind of iron wire as before (0·79 mm. diameter and 33 centims. long) to test the effects of load in various magnetic fields when the wire was in the soft annealed state. The wire was carefully annealed and surrounded by two solenoids, in one of which a current was maintained which neutralised the vertical component of the earth's field, while the other was used to give the desired magnetising force. The wire (whose limit of elasticity was pretty sharply defined at 10 kilos.) was never loaded with more than 6 kilos., and instead of reducing the load to zero in each cyclic application, a load of 1 kilo. was kept always on, in order to avoid errors due to local bending and unbending, which are much more liable to occur when we are dealing with annealed wire than with wire which has been well straightened by stretching. The process of experiment was in other respects the same as before. Before the application of each magnetising field the wire was demagnetised by reversals (with 1 kilo. on), then the field was applied, and then a cycle of loads 1—6—1 was applied several times while the magnetic changes were noted. Except in the region of approximate saturation, the first effects of loading were positive, but in succeeding applications *on* gave diminution and *off* gave increase of magnetism, the curves for *on* and *off* forming a simple loop like that already described in speaking of the effects of stress on an annealed wire exposed to the vertical component of earth's field. (§ 78.)

The action was in this case examined in fields ranging up to 12 c.g.s. units, and the results are shown in Plate 64, fig. 43, where the curves are numbered as follows :—

Curve.	Magnetising field (in c.g.s. units.)
I.	0·68
II.	1·50
III.	2·05
IV.	2·73
V.	3·41
VI.	4·78
VII.	12·0

The line which (in every case but curve VII.) slopes up to the right starts from the value of magnetism reached by applying the field to the (previously demagnetised) wire, and shows the effect of the first loading. The cycle of loads was not applied often enough to make the changes of magnetism cyclic, except at the highest magnetisation, where (as in the stretched wire already referred to) a cyclic state is reached very quickly. In each field the effects of several successive loadings and unloadings are exhibited in fig. 43, in a way which inspection of the figure will make obvious. Curve I*a* was obtained in the field 0·68 by tapping the wire and then applying and removing loads.

Each kilo. of load corresponds in this wire to a stress of 2·04 kilos. per sq. mm.

§ 90. *Curves of* $\mathfrak{I}$ *and* $\mathfrak{H}$ *taken under Constant Load.*—Concurrently with the series of experiments described in §§ 87–88, and represented in figs. 41 and 42, another series of observations was conducted on the same wires and with these in the same states as in the foregoing tests, by a wholly different method. Each wire, after being demagnetised by reversals, had a definite load applied to it. Then magnetising force was gradually applied, and the relation of $\mathfrak{I}$ to $\mathfrak{H}$ was observed. This gave a curve of magnetisation under that particular load. Then, after demagnetisation, a different load was put on, another curve of magnetisation taken by again applying magnetising force, and so on. The magnetising force was applied gradually and continuously by means of the slide described in § 18, and the magnetisation was observed by the direct magnetometric method. In fact, as regards measurement of magnetism and field, the conditions of each experiment were identical with those under which the changes of magnetism were caused by changing loads in a constant field, and the same factors were used to reduce to absolute measure the results of these two methods of experiment. The results of the two are, therefore, directly comparable with each other.*

§ 91. *Annealed Iron Wire.*—In the group which I shall first describe of observations made in this way the wire of § 87 was used, but in the *soft annealed* state which preceded the stretching by 15½ kilos. there described. Curves of the relation of magnetisation to magnetising force were taken (May 1, 1882) with loads of 1, 2, 4, and 6 kilos., as well as with no load, and after each curve taken with load, the load was removed and a curve without load taken, in order to see whether the loading had produced any permanent change in the quality of the metal. For brevity a curve of $\mathfrak{I}$ and $\mathfrak{H}$, taken with no load on the wire, will be called a "normal" curve. The normal curves after each unloading agreed so very approximately with one another as to show that loading up to 6 kilos. did not sensibly harden the wire. It should be remembered that as the diameter was 0·77 mm. 1 kilo. of load corresponds to a stress of 2·15 kilos. per sq. mm. After each magnetisation the magnetising current was gradually reduced to zero, observations of $\mathfrak{I}$ being made; the wire was then unloaded and demagnetised by reversals, a new load applied, and another curve of magnetisation taken.

The results are shown in Plate 64, fig. 44, where $\mathfrak{I}$ and $\mathfrak{H}$ are given in absolute units. The curves show the effects of gradually removing as well as gradually applying magnetising force, the effects of removal of $\mathfrak{H}$ being shown, however, only towards the end of that operation, in order not to confuse the figure. The general characteristics of the curves are these:—For equal *low* values of the magnetising force the magnetisation is greater in the loaded than the unloaded curves. But each loaded curve crosses the normal or unloaded curve at a value of $\mathfrak{I}$ which is lower the higher the stress, and beyond that the magnetic susceptibility is less when the wire is under load than when it is not loaded. Finally, in the region of saturation, all the curves are converging, and it appears probable enough that with very great magnetising forces the magnetisation would be the same for no load and for all values of the load.

As regards the *off* curves, these preserve the relative places the *on* curves have in the region of saturation, that is to say, the normal curve is highest, and the curves for load lie lower the greater the load is. The distance between the *off* curves augments as the magnetising force approaches zero, and we then find that the loaded wire is much less retentive than the unloaded.

The maximum value of κ or $\frac{\mathfrak{I}}{\mathfrak{H}}$ is greater in the 1 kilo. curve than in any of the others. But the steepest slope or $\frac{d\mathfrak{I}}{d\mathfrak{H}}$ is to be found in the normal curve.

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* A few curves of magnetisation under load, similar to those about to be described, have recently been published by Mr. R. SHIDA, in a paper entitled "Experimental Determinations of Magnetic Susceptibility and of Maximum Magnetisation in Absolute Measure," Proc. Roy. Soc., No. 227, 1883.

The presence of stress in an annealed wire tends to round off the outlines of the curve of magnetisation, so that it resembles somewhat the curve described in § 33 as characteristic of a wire which has been stretched beyond its limit of elasticity, and this happens although the stress is too small to give the wire any permanent set, or to harden it appreciably.

§ 92. Plate 64, fig. 45 represents the results of the same experiment in a different manner. In it the values of magnetisation reached by applying a particular magnetising force while the load was kept constant, are plotted in relation to the constant loads (2, 4, and 6 kilos.) which were kept on during the several magnetisations, and a curve is drawn through the points so obtained. This gives a series of curves, each referring to some particular value of the magnetising force, which is entered in the figure alongside of the curve. The curves may be said to represent the relation which the susceptibility to magnetisation (by the several assigned magnetising forces) bears to the stress upon the wire. They are closely analogous to curves of the type of figs. 31 to 43, but with this important distinction, that in the present case the load was on before the magnetising field was applied, while in them the magnetising field was applied first and then the load. Were there no hysteresis in the changes of magnetisation the curves of fig. 45 should agree exactly with curves obtained by applying to an annealed wire a constant magnetising force first and then varying the stress. But we know that there is hysteresis in the relation of magnetisation to magnetising force, under constant load. We also know that there is hysteresis in the relation of magnetisation to stress, when stress is changed in a constant field. There is, therefore, a double reason why the curves obtained in the way now exemplified should differ from those of fig. 43, where, as here, the iron was dealt with in the soft annealed state.

Fig. 45 shows that at low magnetisations the susceptibility is increased by the presence of longitudinal pull. At higher magnetisations we find a maximum of susceptibility, which occurs with lower and lower values of the stress the stronger the magnetisation is, until finally, when we approach the region of saturation this maximum disappears, as it were, to the left of the figure, and we then find the susceptibility greater without than with the load.

This is quite in agreement with the results already obtained by the other method, —the method, namely, of varying stress in a constant field (see figs. 36–38, and 43, §§ 76–82, 89).

§ 93. *Stretched Iron Wire.*—The effects of the presence of stress on magnetic susceptibility become much more conspicuous when we are dealing with wire which has been stretched beyond its elastic limit.

The wire of the above experiment was stretched by applying a load of 15½ kilos., which brought it to the state already mentioned in § 87. In addition to the experiments of the old type described there, observations were made upon it (May 4, 1882) by the plan of magnetising under constant load. The loads used were 0, 1, 2, 4, 6, and 8 kilos. After demagnetising by reversals the load was put on, then magnetising force was gradually applied up to 18 c.g.s. units and gradually removed, while magnetometer observations were taken. The results are shown in Plate 64, fig. 46, where, for the sake of comparison, a normal (no-load) curve, taken just before the wire was drawn by $15\frac{1}{2}$ kilos., is also given. The effects of load, especially of 6 or 8 kilos., is enormous in increasing the susceptibility in the first part of the curve: the susceptibility is actually *eight times* greater with than without that load in the early part of the magnetisation. It is particularly interesting to notice that the wire after being stretched and then loaded with 4, 6, or 8 kilos. is actually *more* susceptible to magnetisation by low fields (under $\mathfrak{H}=1$) than it was when in the soft annealed state. The crossing of the 6 and 8-kilo. curves will be noticed.

§ 94. Neither the range of loads nor the greatest magnetising force applied was sufficiently great to make this experiment exhaustive, and accordingly another series of similar observations was made (May 10, 1882) on the same wire after it had been further drawn by 18·5 kilos. Its state was then identical with that in which the experiment of § 88 was made, and the results given below are, therefore, directly comparable (allowance being made for the effects of hysteresis) with those there described as obtained by the other method, and exhibited in fig. 42. Curves of the relation of $\mathfrak{J}$ to $\mathfrak{H}$ were taken under the following constant loads:—0, 1, 2, 4, 6, 8, 10, 12, 14, 16·2, 18·5 kilos., the highest magnetising force reached being 33·6 c.g.s. units. Before taking each curve the previous magnetisation was removed by the method of reversals before the previously applied load was removed. Then that load was removed, and the process of demagnetising by reversals was repeated. Then the desired load was put on, and the wire was magnetised, the relation of $\mathfrak{J}$ to $\mathfrak{H}$ being observed. The results are given numerically below, and are shown in Plate 64, fig. 47, from which it will be seen that the curves taken with load, although lying much higher than the normal or no-load curve in the early part of their course, begin to cross it as the magnetisation becomes strong, the first to cross being the 18·5-kilo. curve—that of greatest stress. In other words, the magnetic susceptibility, although far greater in the stressed than the unstressed wire so long as the magnetisation is moderate, becomes less in the stressed wire as the region of saturation is reached. During nearly the whole of its length the 8-kilo. curve lies higher than any of the others. It is with this load that the *maximum* of susceptibility occurs, except at very low and again at very high values of the magnetisation.

In the lower part of the same figure curves are given which show the relation of $\mathfrak{J}$ to $\mathfrak{H}$ during the gradual removal of $\mathfrak{H}$.

MAGNETISATION of a stretched iron Wire under various (constant) loads.—Plate 64, fig. 47.

$\mathfrak{H}$.	Values of $\mathfrak{I}$ reached by magnetising under the following loads :—											
	0.	1 kilo. or 2·3 kilos. per sq. mm.	2 kilos. or 4·6 kilos. per sq. mm.	4 kilos. or 9·2 kilos. per sq. mm.	6 kilos. or 13·8 kilos. per sq. mm.	8 kilos. or 18·4 kilos. per sq. mm.	10 kilos. or 23·0 kilos. per sq. mm.	12 kilos. or 27·6 kilos. per sq. mm.	14 kilos. or 32·2 kilos. per sq. mm.	16·2 kilos. or 37·3 kilos. per sq. mm.	18·5 kilos. or 42·5 kilos. per sq. mm.	0 repeated.
0	0	0	0	0	0	0	0	0	0	0	0	0
1·15	12	15	20	35	38	..	62	55	45	44	42	11
2·01	27	32	48	92	112	160	163	138	118	110	99	25
2·87	50	62	95	192	250	310	310	258	225	200	180	45
4·31	110	150	215	377	490	542	532	470	425	380	343	108
5·75	195	258	342	525	650	698	688	620	570	522	475	190
7·19	282	362	455	637	760	800	785	725	675	620	572	280
8·62	367	455	553	725	842	875	860	800	755	700	650	365
11·50	525	612	710	860	955	978	962	908	870	813	762	520
14·37	660	748	835	958	1032	1045	1032	982	947	892	845	655
17·25	780	858	933	1030	1088	1092	1080	1038	1005	955	910	770
20·12	882	950	1012	1088	1128	1128	1118	1078	1048	1003	960	873
23·10	970	1025	1078	1130	1160	1155	1145	1107	1085	1042	1000	958
26·0	1048	1088	1130	1165	1185	1178	1168	1133	1112	1072	1035	1035
33·6	1182	1200	1225	1229	1232	1220	1210	1180	1168	1135	1105	1180
23·10	1112	1142	1172	1190	1200	1188	1175	1145	1127	1088	1052	1108
17·25	1028	1078	1120	1158	1175	1162	1150	1112	1090	1045	1002	1023
11·50	885	958	1023	1095	1142	1125	1108	1060	1028	973	920	882
5·75	695	777	862	982	1047	1045	1018	955	905	840	788	690
2·87	585	662	755	890	970	970	935	863	808	745	690	580
0	465	537	622	762	840	838	793	712	660	603	550	455

§ 95. The same results are also exhibited graphically in Plate 65, fig. 48, in the manner explained in § 92, and this figure admits of immediate comparison with fig. 42, because both figures refer to the same piece of wire in the same physical condition. Fig. 48 shows very clearly how there is, for each value of the magnetising force, an amount of stress for which the susceptibility is a maximum, how this maximum occurs at a lower and lower value of the stress as the magnetisation is increased, and, finally, how the resultant effect which the greatest load (18·5 kilos.) exerts on the susceptibility passes from positive to negative as the state of saturation is approached. The magnetising force to which each of the curves relates is given in the figure, and is in several values the same as that at which, in the previous group of observations, the effects of loading and unloading were investigated (§ 88). From what has been said in § 92 it will be at once seen that the curves of fig. 42 would agree with those of fig. 48 if the effects of hysteresis were absent. In that case it would be a matter of indifference whether stress were applied before magnetising force, or magnetising force before stress. Hysteresis affects both experiments, though quite differently, but by examining the two sets of curves together we can see the general relations of magnetic susceptibility to stress in the features which are exhibited by both. From the two together we may confidently conclude that the influence of pull on magnetism is positive until the magnetisation is so much raised as to bring about the VILLARI reversal; that this positive effect is slight at low magnetisations, and increases greatly so as to pass a maximum before reversal; that the amount of this positive effect depends on the amount of stress applied, passing a maximum whose position varies in the way stated above; that the value of the magnetisation at which the VILLARI reversal occurs depends on the value of the stress, being lower the greater the stress. In fact, if we deal only with very small stresses it is doubtful whether any reversal of the positive effect of stress would be reached even at the highest attainable value of the magnetisation.

§ 96. *Residual Effects of Stress Changes occurring when the Wire is free from Magnetism.*—During the progress of some experiments of the same type as those just recorded, but preliminary to them, a very curious phenomenon was noticed, which gave a fresh and interesting example of the presence of hysteresis in a somewhat occult form.

If we take an iron wire which has already been stretched beyond its original limit of elasticity, and which has, therefore, reached a stable state as regards any subsequent applications of lesser pull, we may load and unload and magnetise it as we please, but provided we demagnetise it by the method of reversals after removing the load, we shall, on taking a new "normal" curve of magnetisation, always find its susceptibility sensibly the same. But suppose that after the wire has been completely demagnetised we apply a load and remove it. During this process there has been no magnetisation visible, and there has been no mechanical change of any ordinary kind. Nevertheless, a molecular change of a very decided character has occurred, which is at once detected when (after applying and removing load) we take a curve of magnetisation. We find the magnetic susceptibility now changed from what it was. The difference of condition is merely this, that in the former case, after removing a previously applied load the wire was passed through the process of demagnetising by reversals before a new curve was taken, while in the latter case the wire was demagnetised before the load was removed, and the load then removed before the curve was taken. So slight a difference of procedure as this might fairly be expected to have no sensible influence on the results of the test, but it was only after recognising that this difference and others like it had an extraordinarily great influence that I was able to obtain the mutually consistent results stated in the above paragraphs, by adhering to a strictly uniform mode in the taking of every curve. I found that after the wire had been demagnetised completely, the application and removal of any load, though producing no immediately visible effect, affected most materially the subsequent behaviour of the wire. Indeed the curve of magnetisation under any given load depends not only on the load actually present, but on any changes of load which have been permitted to take place since the preceding demagnetisation. For example, if a curve was to be taken with (say) a load of 3 kilos. on the wire, and if, after complete removal of all visible magnetisation, the load were accidentally raised to 4 kilos. and 1 kilo. then removed, the resulting curve was very sensibly different from what it would have been if the weight had simply been raised to and kept at 3 kilos., and this, too, in spite of the fact that the wire had been frequently subjected to more than four times that amount of stress, and was therefore in a mechanically stable state.

§ 97. This obscure but very interesting phenomenon of the residual effects of previously applied stress formed the subject of a large number of experiments, made for the most part on the wire of § 94, which had been previously stretched by a load of 18·5 kilos. In describing these experiments, the interest of which lies only in the comparative and not in the absolute values of the induced magnetism, it is needless to reduce the observations to absolute measure, and I shall generally give merely the galvanometer and magnetometer readings, instead of $\mathfrak{H}$ and $\mathfrak{I}$.

(May 12, 1882.) After many applications of a load of 18·5 kilos. the wire was demagnetised by reversals without load, and then on applying magnetising force the readings in Column I. below were taken.

Next, after removal of the load, the wire was again demagnetised, then loaded to 18·5 kilos., then unloaded, and the observations of Column II. were taken. Lastly, the wire was again demagnetised, loaded to 18·5 kilos., unloaded, *tapped*, and the observations of Column III. taken. In all three cases there was no load on the wire during the process of magnetisation.

Galvanometer readings.	$\mathfrak{H}$.	Magnetometer readings.		
		I. After demagnetisation with no load.	II. After the cycle 0 – 18½ – 0.	III. After the cycle 0-18½-0 and then vibration.
0	0	0	0	0
20	1·15	5	8	5
35	2·01	11	19	10
50	2·87	19	40	17
75	4·31	44	73	35
100	5·75	78·5	110	70
150	8·62	149	176	150
200	11·50	212·5	230	214
250	14·37	267	278	268
300	17·25	314·5	321	314
350	20·12	355	358·5	354
400	23·00	390	394	388
450	25·87	420	420	422
576	33·12	472	472	471
400	23·00	445	441	443
300	17·25	412	407	409
200	11·50	355	351	352·5
100	5·75	278	275·5	276
50	2·87	234	232	232
0	0	183·5	183	182

The same experiment is shown in the curves of Plate 65, fig. 49, where the dotted curve is No. III., taken after applying the cycle 0—18·5—0 to the previously demagnetised wire, and then vibrating it before magnetising, the full line (I.) is the curve taken after demagnetisation with no load, and the broken line (II.) is the curve taken after the cycle of loads 0—18½—0. No vibration occurred during the taking of the curve.

This experiment (which was confirmed by many others like it) shows that the application and removal of load, after the wire is demagnetised, increases its susceptibility to subsequent magnetisation, but that this increase of susceptibility is removed, and the original condition approximately restored, by vibrating the wire after the application and removal of load, and before beginning to magnetise. Curve No. III., nearly coincides with No. I., proving that the residual effect of the previous loading and unloading, which had raised the susceptibility to the level shown by Curve II., was neutralised in great part by mechanical disturbance.

§ 98. Now it seems from this, that if we apply and remove stress in a wire whose magnetic state is entirely neutral, *we cause some kind of molecular displacement in the relation of which to the applied stress there is hysteresis.* This molecular displacement, whose precise character has yet to be determined, is the cause of the changes of magnetic susceptibility which accompany changes of stress.

When a load is applied and removed the molecular displacement lags behind the changes of stress, and we therefore find on subsequently magnetising the wire that the susceptibility has been changed by the process—that its value is more nearly equal. than before to what it would be with a load actually on the wire. In the present case, to apply and remove 18½ kilos. has raised the magnetic susceptibility above the value it had after the wire was demagnetised by reversals. The action is, in fact, analogous to the augmentation of magnetism which occurs (through the influence of hysteresis) in a *magnetised* wire which has been tapped, by applying and removing a load (see §§ 78, 85).

Here there is no visible magnetism for the stress to act on, but we find evidence of the fact that a load has been applied and removed in the augmented state of susceptibility in which it leaves the wire.

Moreover, just as in the more obvious manifestations of hysteresis already dealt with, the effect of vibrating is to remove in great part the traces of previous operations. The augmented susceptibility, which is found after loading and unloading, disappears if we vibrate the wire before magnetising, just as when a wire hanging in a magnetic field is vibrated and then loaded and unloaded, the abnormally raised value of the magnetism thereby reached is reduced by again vibrating (§ 85).

§ 99. I was thus led to conceive of the magnetic susceptibility of iron—a quality measurable only by magnetising, but easily thought of as existing apart from any actual magnetisation—as changing with changing stress in a manner which involves hysteresis, even when the changes of load take place in the absence of all visible magnetisation. Its value depends on previous as well as on actually present loads. Now the general relation of susceptibility to stress, as tested by magnetising under constant load, resembles that of magnetism to stress when the stress is changed in a constant field (*cf.* figs. 48 and 42), and therefore it seemed probable that the changes of susceptibility which occur when we load and unload a demagnetised wire (and which we now know to be characterised by hysteresis) would, if we could render them visible, be found to follow curves very much like those of figs. 31-42. In those curves we found a very notable difference between values reached with, say, 3 kilos., on the *on* and *off* branches, and this led me to expect a similar kind of difference in the susceptibility of a wire when, after demagnetisation, it was treated in these two ways :—

(1) Load to 18·5, and unload to 3.
(2) Load to 18·5 ; unload to 0 ; load again to 3.

The susceptibility in the first case was likely to be *greater* than in the second, except perhaps when the magnetism approached saturation. Further, it seemed reasonable to expect that the difference between these cases would be very nearly eliminated if in each case the wire was vibrated before the curve of magnetisation was taken.

§ 100. That these expectations were completely fulfilled the following experiment (of May 13, 1882) will show. The wire used was the same as before. Curves of magnetisation were taken after the several modes of treatment set forth in the following table, which also gives the recorded observations of current and magnetism :—

Galvanometer readings. (To reduce to $\mathfrak{H}$ multiply by 0·0575.)	Magnetometer readings.				
	I. Demagnetised with no load. Then no-load (normal) curve.	II. Demagnetised with no load. Then 0 – 18½ – 3. Load = 3 kilos.	III. Demagnetised with no load. Then 0 – 18½ – 0 – 3. Load = 3 kilos.	IV. Demagnetised with no load. Loaded to 18½, unloaded to 3 kilos., and *tapped* before magnetising. Load = 3 kilos.	V. Demagnetised with no load. Loaded to 3 kilos. and *tapped* before magnetising. Load = 3 kilos.
0	0	0	0	0	0
25	6½	22	13	11	10
50	20	70	14	36	34
75	45	139	109	103	100
100	79	198	176	174	168
125	..	242	226	227	219
150	150	276	265	268	259
200	212	328	323	328	320
250	262	..	365	369	365
300	..	398	398	403	400
350	354	424	425	429	427
450	420	461	462	467	466
588	477	491	494	499	498
0	184	274	275	277	276

Curves corresponding to cols. I., II., and III. of the above table are given in Plate 65, fig. 50. The table and curves show that the susceptibility, except when the magnetisation is high, is greater with a load of 3 kilos., reached by unloading from 18½, than with the same load reached by loading from zero. But columns IV. and V. show that when, after each of these processes, we subject the wire to vibration before beginning to magnetise (it must not be supposed that there was any vibration during magnetisation) this difference is so much reduced as to be scarcely sensible. The effect of tapping is, so to speak, to blot out the recollection of previously existing stresses, and to make the susceptibility depend only on the actually present load.

In addition to the observations stated above, a magnetisation curve was taken after the operation 0—18½—0—18½—3, but this, as might be expected, was almost absolutely coincident with No. II. A repetition of the conditions of col. III. gave sensibly the same readings as those of col. III.

In this experiment 3 kilos. was selected as the load under which the wire should be tested, because in the curves of fig. 42 the difference between the *on* and *off* curves is on the whole greater for 3 kilos. than for any other load.

§ 101. In the experiment of the last paragraph the magnetising solenoid was wound directly on the iron wire, and it seemed conceivable that the wire might have been grasped by it so closely as to be prevented from fully responding throughout its whole length to the change of stress caused by loading and unloading. I did not think it possible that this could have occurred to such an extent as to account for the hysteresis of susceptibility with regard to load which had been observed, but to place the matter beyond doubt, this solenoid was unwound, and another was applied which, instead of being wound on the wire itself, was wound on a glass tube inside of which the wire hung free.

This new set of conditions gave results which entirely confirmed those obtained before, and which may therefore be described only very briefly. The magnetising force was not raised so far as before, as the early parts of the curves were of principal interest.

The results of four series of readings are shown in Plate 65, fig. 51, as follows :—

I. is the normal or no-load curve, taken after demagnetising with no load on.

II. is the curve taken after first demagnetising with no load and then performing the cycle 0—15 kilos.—0 before magnetising.

III. is the curve taken with 3 kilos. on after first demagnetising with no load, and then applying the loads 0—10—3 before magnetising.

IV. is the curve taken with 3 kilos. on after first demagnetising with no load, and then applying the loads 0—10—0—3 before magnetising.

The same differences are exhibited as in the former instance. II. shows greater susceptibility than I., and III. than IV. It is indeed not a little remarkable that such apparently trifling variations of conditions should cause the great differences which these curves manifest.

§ 102. With the same wire and the glass-tube solenoid mentioned in the last paragraph the following additional observations were made.

The wire was demagnetised by reversals, with no load, and then loaded with 14 kilos. and a curve taken. The wire was then demagnetised by reversals *with the* 14 *kilos. on*, and a second curve taken. Then the load was removed, and similar pairs of curves taken with 12, 10, 8, 6, 4, and 2 kilos. of load. Plate 65, fig. 52, shows the results. In the left-hand portion of the figure we have the curves taken with loads of 14, 12, 10, 8, 6, 4 and 2 kilos., when each of these loads was applied after demagnetisation, and no second demagnetisation took place before beginning to take the curve. In the right-hand portion of the figure the conditions are the same except that after the load was put on a second demagnetisation was performed before the curve was taken.

Each curve in the latter series lies below the corresponding curve in the former, showing that demagnetisation after loading leaves the wire in a less susceptible condition than loading after demagnetisation produces. Comparing the curves of each group amongst themselves we see that with both modes of treatment a stress of 8 kilos. gives, throughout the greater part of the curves' course, a greater value to the susceptibility than any of the other stresses. It should be remembered that each kilo. of total load means an intensity of stress equal to 2·3 kilos. per sq. mm.

§ 103. The parts of these curves near the origin lie so close to one another that their relative positions are not clearly shown in fig. 52, and the results of this experiment are better seen by plotting the same observations in the manner described in

§ 92 and already exemplified in figs. 45 and 48, where the magnetisation reached under each load is plotted in terms of the load. This is done in Plate 66, fig. 53, the full lines of which refer to the observations made when the wire after being loaded in a magnetically neutral state was not again subjected to the process of demagnetising by reversals before its susceptibility was tested, and the dotted lines refer to the corresponding observations made when the wire, after being loaded, *was* again subjected to the process of demagnetising by reversals. The two curves (plain and dotted) start of course from the same point (at the zero of load), but within the range of magnetisation and load here examined each dotted curve lies below the corresponding plain one; in other words the process of demagnetising when loaded reduces the susceptibility. The load giving *maximum* susceptibility is slightly different in the two, being greater in the dotted than in the plain, and the range of difference of susceptibility under different loads is strikingly less in the dotted than in the plain curves, especially when the magnetisation is feeble. The numbers on the right of the plain curves show the magnetising force corresponding to each pair.

§ 104. Up to this point the experiments on the residual effects of previous stress on susceptibility to subsequent magnetisation had all been made on iron wires which had been hardened by stretching beyond their elastic limit. It seemed desirable to examine in the same way a soft annealed wire, and accordingly the one already used in § 89 was also tested (May 24, 1882) by applying magnetising force while the load was kept constant. The glass-tube solenoid (§ 101) was again used, and the load was not allowed to exceed 6 kilos., nor to be less than 1 kilo., as in § 89. Plate 66, fig. 54, shows the curves of magnetisation obtained in the following four conditions:—

First pair.
I. Demagnetised with 1 kilo. on; tested with 1 kilo. on.
II. Demagnetised with 1 kilo. on; loaded to 6 kilos. and unloaded to 1 kilo.; tested with 1 kilo. on.

Second pair.
III. Demagnetised with 1 kilo. on; loaded to 6 kilos. and again subjected to the process of demagnetising by reversals with 6 kilos. on; then tested with 6 kilos. on.
IV. Demagnetised with 1 kilo. on; loaded to 6 kilos. and the cycle 6—1—6 performed three times, ending with 6 kilos.; then tested with 6 kilos. on.

One kilo. of load gives a stress of 2·04 kilos. per square mm.

We find that the 6-kilo. curve No. III. lies at first higher and afterwards lower than the 1-kilo. curve No. I.—a result which is entirely in agreement with the conclusions deducible from experiments made by changing loads in a constant magnetic field. We find also that the two curves in each pair differ, in virtue of the residual effects of previous operations, in the manner which previous experiments lead us to expect—namely, that the process of applying reversals after the load is on reduces susceptibility.

It is interesting to notice (as was shown by the curves of figs. 44 and 46, &c., and is now confirmed here) that whereas the presence of a moderate stress, such as 12 kilos. per square mm., greatly increases the susceptibility of a previously stretched wire, it greatly reduces the susceptibility of a soft annealed wire, although, as is evidenced by the recovery of susceptibility after the stress is removed, the stress is too small to produce any sensible hardening of the metal.

§ 105. I have now shown that both in the soft annealed and in the hard-drawn state iron exhibits variations of magnetic susceptibility under varying loads which depend not merely on the existing state of stress, but on the states of stress which have preceded that which is selected as a condition of experiment, although these preceding stresses produce no permanent mechanical effects, and, indeed, leave no directly visible traces of their having been applied, and although the piece under test is free from visible magnetism during these changes of stress. But, with respect to this last point, it occurred to me that the conditions of the foregoing experiment were not altogether satisfactory, for this reason, that the wires were to some extent affected by the horizontal component of the earth's magnetic force. They hung vertically, and the vertical component of the terrestrial field was neutralised by a solenoid provided for that purpose. The only *longitudinal* magnetisation was therefore that given by the other or magnetising solenoid. But this arrangement left the horizontal component of the terrestrial field to act *transversely* on the wires, producing, of course, some small amount of transverse magnetisation, which must have been altered with every change of load. Now it was conceivable, though it appeared very unlikely, that the alterations of this *transverse* magnetism produced by changes of load should (even in the absence of all longitudinal magnetism) produce changes of susceptibility to subsequent longitudinal magnetisation sufficiently considerable to account for the phenomenon discussed in the preceding paragraphs (§§ 96–104). For this reason I judged it desirable to repeat some of the above experiments under conditions which would make any action of the kind impossible. Accordingly the wire to be magnetised was suspended along the line of dip, and the current in the neutralising solenoid was increased to the value necessary to make it neutralise the full terrestrial field. The wire was now free from all magnetising force, and, after being completely demagnetised, it was subjected to stress as before. It was then found that the peculiar residual effects of stress described in §§ 96–104 were as conspicuous as before. The susceptibility depended, just as much as when the wire was in the usual vertical position, on preceding as well as on actual states of stress. The experiment showed conclusively that the transverse magnetism which must have existed in the wire when set vertically did not materially affect the results, and could not have been the cause of the residual differences of susceptibility which were found when a wire, free from longitudinal magnetism, was brought to a given state of stress through different preceding states.

§ 106. *Magnetisation under constant Loads in a high Field.*—It appeared desirable to examine the form which curves of magnetisation under constant load took when the field was raised to values considerably greater than those of preceding experiments of this type, chiefly to see whether with very strong magnetising forces the curves became convergent.

A specimen of the iron wire usually employed (0·79 mm. in diameter and 33 cm. long), was annealed and subjected to a magnetising force which was gradually increased to 97·9 units, first with a load of one kilo. on it, and afterwards (after demagnetisation by reversals) with a load of 6 kilos. The results are given in the table below, reduced to absolute measure. The same wire was then stretched (in a non-magnetic state, and in a neutral field) by applying a load of 15 kilos., giving an elongation of 2·2 cm., and was again magnetised, successively in two different conditions, namely, with no load and with a load of 8 kilos. The load of 8 kilos. was selected because previous experiments had shown that with moderate magnetising forces the susceptibility is greater under 8 kilos. than under any other load. At the same time the earlier experiments had shown that as the magnetising force was increased, the load giving maximum susceptibility became less and less, and therefore had made it likely that the 8-kilo. curve would be found to cross and run under the normal or no-load curve when the magnetising force was greatly heightened. The results of the present experiment, which are stated below, confirmed this expectation.

MAGNETISATION of Iron in a strong field under constant loads, Plate 66, fig. 55.

BEFORE stretching (soft annealed state).

$\mathfrak{H}$.	Load = 1 kilo., or 2·04 kilo. per sq. mm. $\mathfrak{I}$.	Load = 6 kilos., or 12·24 kilos. per sq. mm. $\mathfrak{I}$.
0	0	0
2·3	346	
3·1	595	441
4·2	791	630
6·3	994	833
8·4	1074	945
12·7	1137	1050
20·3	1173	1120
33·6	1207	1169
48·9	1239	1204
60·9	1249	1221
74·7	1260	1232
88·6	1264	1242
97·9	1270	1246
0	1092 (residual)	847 (residual)

AFTER stretching (hardened state).

Load = 0.		Load = 8 kilos., or 17·4 kilos. per sq. mm.	
$\mathfrak{H}$.	$\mathfrak{I}$.	$\mathfrak{H}$.	$\mathfrak{I}$.
0	0	0	0
3·1	56	6·3	600
4·2	97	8·4	735
6·3	142	21·0	1018
8·6	261	42·0	1138
12·9	455	63·0	1190
20·5	724	84·0	1231
33·8	1041	110	1261
50·4	1190		
62·6	1232	0	652 (residual)
77·1	1268		
90·7	1292		
100·6	1309		
0	455 (residual)		

The same two pairs of observations are plotted in Plate 66, fig. 55. The two curves with 1 kilo. and 6 kilos. on the unstretched wire are approaching each other distinctly enough at high values of $\mathfrak{H}$, but this is not so clearly the case in the other pair, and so far as the question of ultimate coincidence is concerned, the experiment is inconclusive. It merely shows that considerable differences are still to be found under magnetising forces of the highest easily attainable value.

At the end of each magnetisation $\mathfrak{H}$ was gradually reduced to zero and the residual magnetism observed. Its value is entered at the foot of each column in the table above. It is curious to notice how much more the residual magnetism is affected by the presence of stress than the induced magnetism is when both have been produced by a strong magnetising force. In the soft wire the final induced magnetism under 1 and under 6 kilos. differs by less than 2 per cent., but the residual magnetism has the widely different values 1092 or 86 per cent. of the induced, and 847 or 68 per cent. The same thing is still more striking in the stretched wire, where indeed the condition of no load gives slightly *more* induced but much *less* residual magnetism than the condition of stress gives. The retentiveness of the soft wire is reduced, and that of the stretched wire greatly increased by the presence of stress during the application and removal of magnetising force.

§ 107. *The Villari Critical Point.*—Allusion has been made (§ 87) to the fact, discovered by Villari, and afterwards by Thomson, that if we take a wire under the influence of a longitudinal magnetising force, and alternately apply and remove a given load, "on" gives augmentation and "off" gives diminution of magnetism if the magnetisation is less than a certain value; but "on" gives diminution, and "off" gives augmentation of magnetism if this value is exceeded. Sir William Thomson calls the value of the magnetising force at which this reversal of effect occurs, the "Villari critical value." It is clear (as he has also pointed out) that the "Villari critical value" depends on the particular amount of load alternately applied and removed, and also upon the mean value of the load.

An examination of my results shows that the effects of stress on magnetism are, on the whole, not greatly different whether the magnetism in question is residual or induced, and I am therefore disposed to regard the Villari reversal as depending rather on the value of the magnetisation than on the value of the magnetising force.

This view receives strong confirmation from experiments which I have described in a paper, "On the production of transient electric currents in iron and steel conductors by twisting them when magnetised, &c."* I have shown there that the inclined pull and push which make up torsional stress give effects on a very strongly magnetised wire which have the same sign as the effects of direct pull and push on a feebly magnetised wire; which is to be explained by the fact that these act on $\mathfrak{I}\cos^2\frac{\pi}{4}$ or only one-half of the whole magnetism, an amount which does not exceed the "Villari critical value" of $\mathfrak{I}$, although one-half of $\mathfrak{H}$ may be made to exceed very largely the value of the magnetising force sufficient to cause the Villari reversal when pull is applied in the direction of magnetisation.

I shall therefore apply the term "Villari critical point" to name that value of the magnetisation $\mathfrak{I}$ at which reversal occurs in the sign of the magnetic difference produced by two (assigned) states of longitudinal stress.

This point may be determined in two ways, which (on account of hysteresis) will lead to different results. We may (with Thomson) cause a repeated alternation to take place from one to the other of the two assigned states of stress, and find the value of $\mathfrak{I}$ at which that alternation causes no magnetic change, or we may take two separate curves of magnetisation (that is, curves of the relation to $\mathfrak{I}$ to $\mathfrak{H}$) in the two assigned states of stress, and see where these curves cross each other. The value of $\mathfrak{I}$ at the crossing point is the critical value for the two given states of stress.

Several of the figures which have been given show values of the "Villari critical point" according to this last mode of definition.

We may, however, conveniently limit the term "Villari critical point" to the case where one of the two assigned loads is zero; that is to say, we may use the term only when we are comparing the magnetic condition reached under a given load with that reached under no load. With this restriction the Villari critical value of $\mathfrak{I}$ is (for any load) that value at which the curve of magnetisation under that load crosses the normal or no-load curve. Thus, in fig. 47 (§ 94) the curves for 12, 14, 16·2, and 18·5 kilos. all cross the normal curve within the range of the observations, and so determine the Villari critical points (as just defined) for these particular loads.

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* Proc. Roy. Soc., Vol. 36 (1883), p. 117.

They show that the VILLARI reversal occurs earlier in stretched iron the greater the stress is, but that even with a stress approaching the breaking strength of the material it does not come till $\mathfrak{I}$ is more than 1000. For loads less than 12 kilos. it would, no doubt, occur at values of $\mathfrak{I}$ greater than those reached in this experiment.

§ 108. In soft annealed iron, however, the VILLARI critical point occurs much earlier, as the experiment of § 91, fig. 44, serves to show. There the curves for 1, 2, 4, and 6 kilos. all cross the normal or no-load curve, at values of $\mathfrak{I}$ which (as in the case of stretched wire) are higher the smaller the load, but which are very much lower than the values of the critical point in stretched wire. The following are the values of $\mathfrak{I}$ and $\mathfrak{H}$ at which the curves with load cross the normal curve in fig. 44 (each kilo. of load gives a stress of 2·15 kilos. per sq. mm.) :—

Load.	$\mathfrak{H}$.	$\mathfrak{H}$.
kilos.		
1	1220	7·3
2	1040	4·3
4	840	3·4
6	690	3·05

Effects of Stress on Retentiveness.

§ 109. In a previous part of this paper (§§ 37–47) I have described experiments by which the retentiveness of iron and steel was examined at various degrees of magnetisation, by applying a given magnetising force, reducing that to zero and observing the residual magnetism, then applying a stronger force, reducing that to zero, and so on. Those experiments showed that the ratio of residual to induced magnetism passes a maximum as the magnetisation is increased.

It seemed desirable to extend the same method of experiment to the case of iron or steel *under longitudinal stress.* Accordingly corresponding observations have been made on pieces of iron (in the soft annealed and also in the stretched condition) and steel, in several different states of stress.

(June 13, 1882.)—A piece of pianoforte steel wire, 31 cm. long, after being annealed, was stretched about 1 cm. by a load of 22 kilos. The diameter after stretching was 0·78 mm. Its retentiveness was then examined (by applying and removing progressively increased values of $\mathfrak{H}$) under four different states of load, namely, 0, 3, 10, and 20 kilos. The process of demagnetising by reversals served to reduce the wire to a magnetically neutral state after magnetisation under each load, and before the next value of the load was applied.

The table below shows the results of this experiment, by giving the observed values of the induced and residual magnetism, and their ratio, for various values of $\mathfrak{H}$:—

PIANOFORTE-STEEL Wire, stretched after annealing, Plate 67, fig. 56.

$\mathfrak{H}$.	Load = 0.			Load = 3 kilos. = 6·27 kilos. per sq. mm.			Load = 10 kilos. = 20·9 kilos. per sq. mm.			Load = 20 kilos. = 41·8 kilos. per sq. mm.		
	$\mathfrak{I}$ induced.	$\mathfrak{I}$ residual.	Ratio.	$\mathfrak{I}$ induced.	$\mathfrak{I}$ residual.	Ratio.	$\mathfrak{I}$ induced.	$\mathfrak{I}$ residual.	Ratio.	$\mathfrak{I}$ induced.	$\mathfrak{I}$ residual.	Ratio.
0	0	0	..	0	0	..	0	0	..	0	0	..
0·52	5	0	0	..	..	..	..	..	..	..	..	..
1·04	11	0	0	12	0	0	12	3	0·25	18	7	0·39
1·56	17	0	0	..	..	..	..	..	..	..	..	..
2·08	23	1·4	0·06	25	4	0·16	32	11	0·34	39	17	0·44
3·12	38	5	0·13	42	8·4	0·20	57	25	0·44	64	31	0·48
4·16	57	11	0·19	67	17	0·25	90	43	0·48	94	48	0·51
5·20	82	23	0·28	90	31	0·34	129	70	0·54	129	70	0·54
6·24	115	42	0·37	126	56	0·44	176	105	0·60	168	98	0·58
7·28	161	76	0·47	174	89	0·51	231	146	0·63	213	126	0·59
8·32	216	120	0·56	232	136	0·59	297	197	0·66	263	162	0·62
9·36	277	170	0·61	300	190	0·63	367	255	0·69	314	199	0·63
10·40	346	229	0·66	371	252	0·68	435	315	0·72	365	235	0·64
11·44	417	291	0·70	..	..	..	..	..	..	..	..	..
12·48	484	353	0·73	524	389	0·74	566	431	0·76	470	314	0·67
13·52	546	409	0·75	..	..	..	619	473	0·76	518	350	0·68
14·56	599	459	0·77	..	..	..	666	517	0·78	554	384	0·69
15·60	648	501	0·77	..	..	..	..	..	..	598	413	0 69
16·64	692	540	0·78	733	582	0·79	747	588	0·79	633	440	0·70
17·68	727	574	0·79	..	..	..	776	622	0·80	664	468	0·70
18·72	760	605	0·796	..	..	..	805	644	0·80	..	..	..
19·76	792	633	0·799	826	666	0·806	829	669	0·81	720	507	0·70
20·80	815	655	0·803	..	..	..	851	690	0·81	739	529	0·71
23·4	875	703	0·804	897	734	0·817	897	728	0·81	788	563	0·71
28·9	955	773	0·809	..	..	..	..	..	..	..	..	..
29·5	..	..	..	987	815	0·826	981	804	0·819	877	636	0·725

The same results are shown in Plate 67, fig. 56, where three groups of curves are given. The lower group shows the induced magnetism, and the middle group the residual magnetism, in terms of $\mathfrak{H}$. The upper group shows the values of the ratio of residual to induced magnetism, also in terms of $\mathfrak{H}$.

The crossing of the curves of $\mathfrak{I}$ (induced) and $\mathfrak{H}$ should be noted. It corresponds, in kind, to the behaviour of iron, and shows that in steel, as in iron, any increase of stress gives first increase and then decrease of susceptibility, according as the magnetisation falls short of or exceeds a certain critical value. Each curve lies at first higher and afterwards lower than curves referring to smaller values of the load.

The middle group of curves shows that the same remark applies to residual magnetism.

Further, the curves which represent the ratio of residual to temporary magnetism in relation to $\mathfrak{H}$ cross each other in the same way. There is thus, for each load, a critical value of the magnetisation, below which the ratio of residual to temporary magnetism is greater, and above which it is less, when the wire is loaded than when it is unloaded (or loaded with a smaller load). For example, under a load of 20 kilos. the retentiveness is much greater than under a small load or no load, so long as the magnetisation is feeble, but less when the magnetisation is strong.

The observations taken with no load confirm what has been already said (§ 44) as to the absence of retentiveness in steel, under weak magnetising forces. But it is very remarkable how the presence of stress increases the retentiveness under weak magnetising forces. A force $\mathfrak{H}$ of 1 unit, for instance, leaves sensibly no residual magnetism in the wire when there is no load, but leaves a residue amounting to nearly 0·4 of the induced magnetism when there is a load of 20 kilos.

§ 110. A piece of iron wire, which, like the last, had (after annealing) been stretched beyond its limit of elasticity, was next examined in the same way (June 14, 1882), the successive states of load being 0, 5, 10, and 14·8 kilos. The diameter of the wire (after being stretched) was 0·72 mm. and its length was 30·5 cm. The load employed to stretch it was 20 kilos.

The induced and residual magnetism of this wire, found under these various states of stress by applying and removing a large number of progressively increased values of $\mathfrak{H}$, are given in the table below, and the same results are shown in Plate 67, fig. 57, where the curves give the relation to $\mathfrak{H}$ of $\mathfrak{I}$ induced, $\mathfrak{I}$ residual, and their ratio :—

IRON Wire, stretched after annealing, Plate 67, fig. 57.

LOAD = 0.

$\mathfrak{H}$.	$\mathfrak{I}$ induced.	$\mathfrak{I}$ residual.	Ratio.
0	0	0	..
0·54	5	..	..
1·35	19	4	0·21
1·62	23	6	0·26
2·16	35	10	0·28
2·70	50	16	0·32
3·24	73	28	0·39
3·78	101	43	0·43
4·32	132	65	0·49
4·86	167	88	0·53
5·40	205	114	0·56
5·94	242	140	0·58
6·48	280	166	0·59
7·02	312	189	0·60
7·56	353	211	0·60
8·10	390	232	0·60
8·64	429	252	0·60
9·18	457	270	0·59
9·72	488	286	0·59
10·26	520	301	0·58
10·80	550	315	0·57
11·88	611	336	0·55
12·96	670	354	0·53
14·04	725	372	0·51
15·17	777	387	0·50
16·20	821	397	0·48
17·55	874	406	0·46
18·90	930	416	0·45
20·25	975	423	0·43
21·60	1014	429	0·42
22·95	1053	432	0·41
24·30	1087	435	0·40
25·92	1123	437	0·39
29·16	1190	439	0·37

Load = 5 kilos., or 12·2 kilos. per sq. mm.

ℌ.	ℑ induced.	ℑ residual.	Ratio.
0	0	0	—
0·54	15	8	0·53
1·08	53	34	0·64
1·62	114	81	0·71
2·16	211	159	0·75
2·62	297	237	0·80
3·24	410	331	0·81
3·78	494	405	0·82
4·32	561	464	0·83
4·86	621	513	0·83
5·40	673	556	0·83
6·48	761	624	0·82
7·56	827	672	0·81
8·64	881	709	0·80
9·72	923	736	0·80
10·80	965	757	0·79
12·15	1004	780	0·78
13·50	1040	795	0·76
14·85	1069	806	0·75
16·31	1095	815	0·74
17·55	1112	822	0·74
18·90	1131	827	0·73
21·60	1164	832	0·72
24·30	1186	835	0·71
26·46	1202	838	0·70
29·16	1225	841	0·69

Load = 10 kilos., or 24·4 kilos. per sq. mm.

ℌ.	ℑ induced.	ℑ residual.	Ratio.
0	0	0	—
0·54	21	12	0·57
1·08	63	39	0·62
1·62	137	94	0·69
2·16	241	176	0·73
2·70	351	267	0·76
3·24	442	351	0·78
3·78	530	416	0·79
4·32	595	468	0·79
4·86	650	513	0·79
5·40	696	547	0·79
6·48	770	601	0·78
7·56	830	637	0·77
8·64	884	664	0·76
9·72	917	685	0·75
10·80	946	696	0·74
12·26	978	712	0·73
13·50	1004	725	0·72
14·85	1028	731	0·71
16·20	1048	739	0·70
18·90	1082	748	0·69
21·60	1105	754	0·68
24·30	1125	757	0·67
27·54	1144	760	0·66
29·16	1154	761	0·66

Load = 14·8 kilos., or 36·1 kilos. per sq. mm.

$\mathfrak{H}$.	$\mathfrak{I}$ induced.	$\mathfrak{I}$ residual.	Ratio.
0	0	0	—
0·54	19	9	0·47
1·08	53	29	0·55
1·62	104	62	0·60
2·16	176	110	0·63
2·70	258	172	0·67
3·78	419	289	0·69
4·86	530	371	0·70
5·99	619	432	0·70
7·02	679	471	0·69
8·10	731	489	0·68
9·45	790	524	0·66
10·80	835	546	0·65
12·26	874	559	0·64
13·50	910	570	0·62
16·20	966	581	0·60
18·90	1011	590	0·58
21·60	1040	595	0·57
24·30	1067	598	0·56
28·08	1093	598	0·55
29·16	1108	598	0·54

Here the results are somewhat different from those of the last paragraph. A distinct maximum occurs in the value of the ratio of residual to induced magnetism in each of the curves with load, as well as in the curve taken without load, and it occurs at earlier values of $\mathfrak{H}$ in the loaded curves than in the unloaded. The effect of stress on retentiveness is again very great, especially with weak magnetisations: thus for $\mathfrak{H}=1$ the ratio of residual to induced magnetism is nearly zero when there is no load, but ranges from ·55 to ·64 in the curves taken with load. It is to be noticed, however, that the curves of ratio and $\mathfrak{H}$, corresponding to 5, 10, and 14·8 kilos. of load, perhaps cross each other at a value of $\mathfrak{H}$ lower than that at which the observations can be said to begin. On the other hand, the curve of ratio and $\mathfrak{H}$ corresponding to no load may perhaps cross the others at a value of $\mathfrak{H}$ much greater than that to which the experiment extends.

The greatest ratio of residual to induced magnetism, throughout the whole group, occurs at the maximum point of the 5-kilo. curve, when its value is 0·83. This is a much greater value than is reached at any point under no load, where the maximum is only 0·6. Moreover, the presence of a moderate load increases, in a still greater proportion, the retentiveness of stretched iron under strong values of $\mathfrak{H}$. At the highest value of $\mathfrak{H}$ which was reached in this experiment the 5 and 10-kilo. curves of residual magnetism, and also of the ratio, lie much higher than the no-load curve.

§ 111. To illustrate this last point more fully the following additional observations were made. The same piece of wire was successively tested under loads of 0, 2, 4, 6, 8, 10, 12, and 14·8 kilos., by first demagnetising it in each case, then applying a force

$\mathfrak{H}$ of 29·16, and then removing that force. The following values of $\mathfrak{I}$ induced and $\mathfrak{I}$ residual were determined in this way :—

Load.	$\mathfrak{I}$ induced.	$\mathfrak{I}$ residual.	Ratio.
0	1157	423	0·37
2	1183	588	0·50
4	1193	757	0·63
6	1189	845	0·71
8	1180	809	0·69
10	1157	757	0·65
12	1138	683	0·60
14·8	1105	598	0·54

From this it will be seen that (for $\mathfrak{H}=29·16$) a load of 6 kilos. gives maximum retentiveness, and that the retentiveness under that load is nearly twice as great as when the same wire is not loaded.

§ 112. The same piece of iron was now re-annealed, and the effect of stress on its retentiveness when in the soft state was examined in the same way (June 16, 1882). The only states of load tested were 0 and 4 kilos. (a load so well within the limit of elasticity as to produce no important permanent mechanical change in the specimen). The results are given below, and also in Plate 68, fig. 58, where, in the lower group of curves, the two full lines show the induced, and the two dotted lines the residual magnetism, while the lines above show the ratio in the same way as before :—

ANNEALED Iron Wire, Plate 68, fig. 58.

Load = 0.				Load = 4 kilos., or 9·76 kilos. per sq. mm.			
$\mathfrak{H}$.	$\mathfrak{I}$ induced.	$\mathfrak{I}$ residual.	Ratio.	$\mathfrak{H}$.	$\mathfrak{I}$ induced.	$\mathfrak{I}$ residual.	Ratio.
0	0	0	—	0	0	0	—
1·08	66	32·5	0·49	0·54	38	21	0·53
1·62	202	141	0·70	1·08	141	94	0·69
2·16	460	381	0·83	1·62	325	242	0·745
2·70	684	601	0·879	2·16	532	419	0·788
3·24	846	767	0·907	2·70	677	543	0·802
3·78	939	860	0·916	3·24	796	640	0·804
4·32	999	920	0·921	3·78	876	705	0·805
5·40	1071	994	0·928	4·37	937	754	0·805
6·48	1109	1024	0·923	4·86	978	787	0·805
7·56	1139	1046	0·919	5·51	1022	816	0·800
8·64	1157	1063	0·919	6·48	1067	856	0·800
9·72	1168	1074	0·919	8·64	1121	891	0·795
10·8	1178	1082	0·918	10·8	1162	913	0·786
13·5	1196	1095	0·916	13·5	1186	926	0·781
16·2	1210	1105	0·913	16·2	1204	933	0·775
18·9	1219	1111	0·911	18·9	1211	939	0·775
21·6	1226	1116	0·910	21·6	1219	942	0·773
25·6	1236	1119	0·905	26·2	1232	946	0·768

The susceptibility is at first greater with 4 kilos. than with no load, but afterwards becomes less with 4 kilos. than with no load (*cf.* § 89). Moreover, the residual magnetism, and also the ratio of residual to induced magnetism, are with low magnetising forces greater, and with high magnetising forces less when there is load than when there is not load.

The ratio of residual to induced magnetism passes a distinct maximum in both cases; but the value of that maximum is greatest in the condition of no load, where it is as great as 0·93. My experiments have presented only one other instance in which the ratio reached so high a figure (*cf.* § 37).

§ 113. The manner in which curves showing the relation of $\mathfrak{I}$ to $\mathfrak{H}$ cross each other, when magnetisation is performed under various constant loads, has been very fully illustrated in a number of the experiments described above. As these, however, with the exception of the test of steel, referred almost exclusively to specimens of iron wire whose mechanical quality was pretty much the same, I judged it desirable to repeat some of the observations, using specimens of iron of very different quality. The results so obtained were in complete agreement with those which have been described above; and for that reason it is needless to repeat them. The general rule, apparently true for any piece of iron is, that the susceptibility, under any one load, is for low values of $\mathfrak{I}$ *greater*, and for high values of $\mathfrak{I}$ *less*, than under a slightly less load, the crossing point occurring at higher values of $\mathfrak{I}$ the smaller the load in question is.

Effects of Temperature on Magnetism.

§ 114. If a piece of magnetised iron or steel be subjected to a cyclic series of changes of temperature, it is well known, from the experiments of WIEDEMANN[*] and others that the changes of magnetism which it undergoes are, in general, not cyclic. If the magnetism which is being dealt with is the residual magnetism left after withdrawal of any magnetising force, then any cyclic process of heating and cooling, or of cooling and heating, results in a fall of magnetism. If, on the other hand, the magnetism dealt with is that which has been reached by the application of a magnetising force that is kept in action during the experiment, then any cyclic change of temperature results in a rise of magnetism. In both cases it is only after any cyclic change of temperature has been many times repeated that the accompanying changes of magnetism become even approximately cyclic.

In this respect the effects of temperature are closely analogous to those of stress (*cf.* § 77). Superposed upon the differential effects of heating and cooling there are progressive permanent changes of the nature of a shaking in of magnetism, when that is induced, or of a shaking out of magnetism, when that is residual.

By repeating any cycle of temperature changes often enough, however, we get rid

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[*] "Galvanismus," II., § 522, *et seq.*

of these progressive changes, and may then study the differential effects of heating and cooling, after the magnetic changes have also become sensibly cyclic.

I have made a number of experiments of this kind with the view of seeing whether, after the magnetic changes become sensibly cyclic, they exhibit static *hysteresis* with respect to the changes of temperature. Since there is clear evidence of such hysteresis in changes of magnetism produced by other means, namely by changes of magnetic field, and also by changes of stress, I expected somewhat confidently to find hysteresis in the relation of magnetism to temperature. But in this expectation I have been altogether disappointed.

§ 115. A long iron wire was fixed inside a vertical glass tube, on which was wound a magnetising solenoid. The tube was connected by india-rubber piping at one end to one or other of three small boilers, capable of supplying a steady current of steam, alcohol vapour, and sulphuric-ether vapour respectively, or to a water-cistern filled with cold water. The other end of the tube led to a water vessel which served as a condenser. An adjacent mirror magnetometer, level with the top of the iron wire, measured the changes of magnetism in the usual way. The method of procedure was this:—

Steam and cold water were alternately passed through the tube many times, until the magnetic state of the wire was observed to change from one to the other of two nearly steady values. Then readings of the magnetometer were taken during the passage through the tube of (1) cold water; (2) ether vapour; (3) alcohol vapour; (4) steam; (5) alcohol vapour; (6) ether vapour; (7) cold water. This completed a cycle of temperature changes in which two intermediate points were fixed during each of the processes of heating and cooling.

This method was adopted in order that the magnetised iron might be exposed sufficiently long to an atmosphere of one definite temperature to give it time to assume that temperature throughout, and so avoid any possibility of error due to the sluggishness with which changes of temperature took place. The stream of vapour was in every case kept up until the magnetometer reading became perfectly steady.

In some experiments, in place of a magnetised wire, a magnetised iron pipe was used, surrounded by a non-conducting jacket, and steam and other vapours were passed through the iron tube itself. This had the advantage that the position of the magnetised iron, with respect to the magnetometer, could be maintained more absolutely constant and unaffected by temperature changes than when the magnet consisted of a wire inside a glass tube. The difficulty of fixing the magnet so that the process of heating and cooling it might not produce displacements whose effects on the magnetometer were comparable with the true change of magnetism, was somewhat formidable on account of the smallness of the whole range of magnetic change caused by alternations between atmospheric temperature and 100° C.

The small range of magnetic change also made it needful to raise the magnetometer to a condition of great sensibility. For this purpose a directing magnet was applied, which neutralised the greater part of the earth's field at the magnetometer needle, leaving only a small directive force, whose intensity was estimated by comparing the period of the needle with its period when this directing magnet was removed. Further, the magnetism of the wire or iron pipe under examination was sufficiently balanced by a second deflecting magnet (arranged to deflect the magnetometer without altering its sensibility) to bring the spot of light on the scale. To determine what ratio the observed magnetic changes bore to the whole magnetism, it was only necessary to observe the whole deflection due to the magnetised wire or pipe, with the magnetometer in an insensitive state, and measure (by comparing periods) the relative sensibility of the two states of the magnetometer.

Much time was spent in tentative experiments before satisfactory results were obtained. These, however, when they were arrived at showed so conclusively that within the limits of temperature experimented on there was no sensible hysteresis in the relation of magnetic change to temperature, that the subject was not pursued at any great length; and for the same reason it will suffice to quote a single set of observations.

§ 116. (November 30, 1882.) In this experiment the magnetising field was the earth's vertical force, in which the iron under test was shaken beforehand. Then water and steam were passed alternately, five times each, after which the following readings were taken. The numbers given are the actual magnetometer readings: by adding 17,000 to them they can be made approximately proportional to the total magnetisation of the piece. The arrows show the sequence of the changes.

Water. (14° C.)		Ether vapour. (35° C.)		Alcohol vapour. (78½° C.)		Steam. (100° C.)
413	→	380	→	302	↘	
						263
	↙	383	←	304	↙	
415						
	↘	381	→	301	↘	
						261
	↙	381	←	304	↙	
415						
	↘	383	→	305	↘	
						262
	↙	382	←	303	↙	
416						
	↘	382	→	304	↘	
						262
	↙	382	←	304	↙	
418						
	↘	383	→	305		

It is clear from these figures that the changes of magnetism between 14° C. and

100° C. are nearly proportional to the changes of temperature,* and exhibit no hysteresis with respect to them. The whole range of change, from 17,415 at 14° C. to 17,262 at 100° C., is only 0·9 per cent. of the total magnetism.

§ 117. The actual amount of magnetic change, and even its sign, depends on the degree of magnetisation. This follows from the known fact that there is a critical value of the magnetising force, or (more probably) of the magnetisation, below which heating increases susceptibility, and above which heating reduces susceptibility. (Baur, Wied. Ann., xi., 1880.) My own experiments have illustrated this in two ways. In some of them the effects have been studied of repeated heatings (up to 100° C.) and coolings, on various degrees of induced or residual magnetism in an iron wire: in others, curves of the relation of $\mathfrak{I}$ to $\mathfrak{H}$ have been drawn when the same iron wire is magnetised, (1) at ordinary temperature, (2) at 100° C. Both methods of experiment have been applied to samples of iron wire both in the soft annealed and hard-drawn states. One or two representative experiments may be quoted.

§ 118. (Jan. 28, 1883.) An iron wire, 1·2 mm. in diameter and 35·7 centims. long, was annealed, and then stretched until its length was 40·5 centims. and its diameter 1·16 mm. In this state it was demagnetised by reversals, and then hung (without mechanical disturbance) under the influence of the earth's vertical force, and very near a magnetometer whose sensibility was raised to a high degree. The wire was surrounded by a tube, round which a magnetising solenoid was wound, and steam at 100° C. and cold water at 6° C. were alternately passed through the tube while the magnetic changes were noted. From time to time, at points which will be indicated below, the total magnetisation of the wire was raised a step, by momentarily setting up in the solenoid a suitable current, which was then interrupted, so that the magnetism dealt with was that part which was retained after the inducing field was reduced to the value 0·34, namely, the earth's vertical force. The following are the values of $\mathfrak{I}$ (expressed as usual in absolute c.g.s. units) which were observed throughout the process.

	$\mathfrak{I}$.	
Original condition of the wire	1·52	
While steam was passing	2·05	Here, in addition to a progressive shaking out and then shaking in of magnetism, there is augmentation of magnetism with steam, and diminution with water. $\mathfrak{I}$ is below the critical point.
" water "	1·98	
" steam "	2·11	
" water "	2·05	
" steam "	2·16	
" water "	2·09	
" steam "	2·20	
" water "	2·12	
" steam "	2·23	
" water "	2·16	

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* The ratio of change of magnetism to change of temperature increases slightly as the temperature rises.

	$\mathfrak{I}$.	
Weak current made and broken raising $\mathfrak{I}$ to	2·51	
Then, while *water* was passing	2·51	
,, steam ,, ...	2·42	
,, water ,, ...	2·34	
,, steam ,, ...	2·44	Still below the critical point.
,, water ,, ...	2·38	
,, steam ,, ...	2·46	
,, water ,, ...	2·41	
,, steam ,, ...	2·51	
,, water ,, ...	2·44	
,, steam ,, ...	2·55	
,, water ,, ...	2·46	
,, steam ,, ...	2·57	
Current made and broken	3·34	
Then, while steam was passing	3·07	
,, water ,, ...	2·99	
,, steam ,, ...	3·07	
,, water ,, ...	2·99	Ditto.
,, steam ,, ...	3·07	
,, water ,, ...	3·01	
,, steam ,, ...	3·10	
,, water ,, ...	3·02	
Current made and broken		
Then, while water was passing	4·03	
,, steam ,, ...	3·63	
,, water ,, ...	3·54	Ditto.
,, steam ,, ...	3·59	
,, water ,, ...	3·52	
,, steam ,, ...	3·56	
,, water ,, ...	3·51	
Current made and broken		
Then, while water was passing	8·38	
,, steam ,, ...	8·36	Just over the critical point.
,, water ,, ...	8·37	
,, steam ,, ...	8·37	
,, water ,, ...	8·38	
Current made and broken		
Then, while water was passing	9·09	
,, steam ,, ...	8·71	

	$\mathfrak{I}$.	
While water was passing	8·71	
,, steam ,, ...	8·67	
,, water ,, ...	8·69	Distinctly over the critical point.
,, steam ,, ...	8·67	
,, water ,, ...	8·69	Steam now gives *fall* and water *rise* of magnetism.
Current made and broken		
Then, while water was passing	9·72	
,, steam ,, ...	9·13	
,, water ,, ...	9·12	
,, steam ,, ...	9·05	Ditto, still more distinctly.
,, water ,, ...	9·09	
,, steam ,, ...	9·04	
,, water ,, ...	9·09	

This, as well as other experiments of the same class, shows that when tested in this way the critical point, at which the sign of the effects of heating and cooling (from 100° to, say, 6° C.) changes, occurs at a very low value of the magnetisation. In the above case it was reached when $\mathfrak{I}$ was about 8, or about the $\frac{1}{150}$ part of the so-called saturation value of $\mathfrak{I}$.

§ 119. (Feb. 5, 1883.) A similar experiment was made with an *annealed* specimen of wire cut from the same bundle as the last, and also 40·5 centims. long.

	$\mathfrak{I}$.	
Original condition	3·94	
While steam was passing	4·80	
,, water ,, ...	4·71	
,, steam ,, ...	4·95	
,, water ,, ...	4·77	Below the critical point.
,, steam ,, ...	4·95	
,, water ,, ...	4·77	
Current made and broken	9·90	
While steam was passing	9·27	
,, water ,, ...	8·88	
,, steam ,, ...	9·03	
,, water ,, ...	8·82	
,, steam ,, ...	9·00	Ditto.
,, water ,, ...	8·79	
Current made and broken	10·89	
While steam was passing	10·08	
,, water ,, ...	9·78	

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While steam was passing	9·81	
,, water ,, ...	9·63	
Current made and broken	11·94	
While steam was passing	11·28	
,, water ,, ...	10·83	
,, steam ,, ...	10·92	
,, water ,, ...	10·74	
Current made and broken	20·04	
While steam was passing	18·63	
,, water ,, ...	18·03	
,, steam ,, ...	17·88	
,, water ,, ...	17·76	
,, steam ,, ...	17·73	
,, water ,, ...	17·61	
,, steam ,, ...	17·58	
,, water ,, ...	17·49	
,, steam ,, ...	17·43	
,, water ,, ...	17·28	Still below the critical point.
,, steam ,, ...	17·31	
Current made and broken	20·55	
While steam was passing	19·26	
,, water ,, ...	18·81	
,, steam ,, ...	18·69	
,, water ,, ...	18·51	
,, steam ,, ...	18·54	
,, water ,, ...	18·45	
,, steam ,, ...	18·42	Ditto.
,, water ,, ...	18·36	
Current made and broken	27·63	
While steam was passing	25·77	
,, water ,, ...	25·20	
,, steam ,, ...	24·90	
,, water ,, ...	24·84	
,, steam ,, ...	24·66	
,, water ,, ...	24·63	Now over the critical point.
,, steam ,, ...	24·45	
,, water ,, ...	24·48	
Current made and broken	41·49	
While steam was passing	41·19	
,, water ,, ...	39·24	

	$\mathfrak{I}$.	
While steam was passing	39·00	
„ water „	38·16	
„ steam „	38·16	
„ water „	37·74	Decidedly over the critical point.
„ steam „	37·89	
„ water „	37·53	
„ steam „	37·65	

Here the critical point appears to be reached when $\mathfrak{I}$ is about 20, but its position is not very sharply defined.

§ 120. The same two wires were also tested by finding, in the ordinary way by the direct magnetometric method, the relation of $\mathfrak{I}$ to $\mathfrak{H}$ during magnetisation: (1) when the temperature was that of the atmosphere, or about 7° C.; (2) when the temperature was maintained at 100° C. by the continuous passage of a current of steam through the tube on which the magnetising solenoid was wound. The following table gives the results, reduced to absolute measure. In each state several successive determinations of the curve of $\mathfrak{I}$ and $\mathfrak{H}$ were made, but as these agreed very exactly with each other, only one set of readings for each state has been reduced:—

EFFECT of Temperature on the Magnetic Susceptibility of Iron Wire, Plate 68, fig. 59.

Soft annealed wire.				Wire hardened by stretching.			
At 7° C.		At 100° C.		At 8° C.		At 100° C.	
$\mathfrak{H}$.	$\mathfrak{I}$.	$\mathfrak{H}$.	$\mathfrak{I}$.	$\mathfrak{H}$.	$\mathfrak{I}$.	$\mathfrak{H}$.	$\mathfrak{I}$.
0	0	0	0	0	0	0	0
1·36	120	1·24	125	2·16	23	2·16	28
1·70	184	1·70	193	3·60	51	3·60	60
2·27	298	2·27	306	7·21	162	7·21	186
2·84	389	2·84	405	10·88	305	10·81	341
3·69	539	3·97	612	14·42	449	14·49	490
4·54	673	5·67	850	18·02	589	18·02	623
5·67	827	7·09	960	21·63	711	21·63	741
7·09	942	8·50	1021	25·37	825	25·23	848
8·50	1012	11·34	1088	29·20	918	28·91	936
11·34	1088	14·18	1119	32·45	983	33·27	1017
14·80	1122	17·01	1140	36·19	1038	36·77	1065
17·01	1145	19·84	1151	39·66	1079	46·36	1150
22·79	1169	22·96	1158	53·21	1206		
30·33	1183	28·01	1160				
0	911 residual.	0	894 residual.	0	363 residual.	0	341 residual.

The same results are also shown graphically in Plate 68, fig. 59, where the full lines are the curves for ordinary temperature, and the dotted lines are for 100° C. Both with the annealed and hardened specimen the 100° C. curves lie at first higher and afterwards lower than the normal curves. The crossing points, at which the effect of this difference of temperature on magnetic susceptibility becomes reversed, occur at values of $\mathfrak{I}$ far higher than the critical value in the former method of experiment. This difference is partly, perhaps, to be ascribed to the fact that here we are dealing with induced magnetism, whereas in the former case the magnetism dealt with was chiefly residual; but the principal reason for the very much higher values of $\mathfrak{I}$ required to be reached here before the effects become reversed is no doubt the hysteresis which exists in the relation of magnetisation to magnetising force. The difference in the results of the two methods is analogous to that which is found when we determine the VILLARI critical point for stress (1) by loading and unloading, (2) by magnetising under constant loads (*cf.* § 107).

§ 121. The range through which magnetism was altered by heating and cooling in the experiment of § 116, and others like it, was so small that I considered it desirable to see whether an increased range of temperature variation would not afford evidence of hysteresis in the relation of magnetism to temperature. With this view, the effect of heating and cooling a steel bar magnet (23 centims. × 2·10 centims. × 0·94 centim.) in a vessel full of oil was investigated as follows:—The bar was hung, from a fixed support above the oil, perpendicular to and with its centre due south of the magnetometer. The temperature of the oil was then altered by means of a lamp, by steps and very slowly, to allow the bar's temperature to be sensibly uniform—and the deflection of the magnetometer was observed. The temperature of the oil was taken by two thermometers, the means of whose readings are given below.

After heating and cooling the magnet until its magnetic changes became nearly cyclic, the following continuous set of observations, extending over 17 hours, was made (Feb. 26–27, 1883). The magnetometer readings are approximately proportional to the total magnetic moment of the bar.

Temperature, deg. C.	Magnetometer.	Hours of observation.	
		h.	m.
8·6*	692	1	15 p.m.
34·6*	656	2	0
48·1	636	3	0
48·4*	635	3	15
67·7	607	3	40
73·3	598	4	10
74·8	596	4	18
75·8	594	4	27
76·1*	593	4	50
91·0	566	5	25
95·2	559	5	45
96·6	556	6	0
98·2	554	6	15
98·8*	552	6	33
113·1	523	7	4
119·8	511	7	20
122·7	505	7	43
123·8	502	8	0
123·8*	502	8	12
151·2	441	8	42
155·3	430	8	56
157·1	425	9	5
158·3	422	9	20
158·4*	422	9	30
Now began to cool.			
131·2	487	10	20
129·8	491	10	40
129·1	495	11	0
129·1*	495	11	10
108·8	538	12	0
105·5	542	12	21 a.m.
104·3*	545	12	45
104·7	546	1	0
103·4	546·5	1	13
79·6	589	2	15
78·4	591·5	2	28
77·4	593	2	35
76·4	593·5	2	45
76·0*	594	2	52
28·2	667	4	10
23·4*	675	4	25
17·8	684	4	45
16·8*	685	4	54
10·8	691	5	35
10·3*	691·5	6	0

The points at which the temperature of the bar was most nearly uniform (marked with an asterisk in the table) have been selected from the above readings and plotted in Plate 68, fig. 60, where the lower line shows the process of heating and the upper line the process of cooling. There is no appearance whatever of hysteresis, in spite of the very considerable amount of magnetic change which the bar underwent.

§ 122. In dealing with a subject such as the magnetisation of iron, to which so much attention has been given, it is difficult to master or to acknowledge adequately the labours of former observers. The comprehensive summary of G. WIEDEMANN, who, both by his own work and by his analysis of the work of others, has contributed enormously to our knowledge of magnetism, and the paper of Chrystal, to which allusion has been made, render some parts of this task at once comparatively easy and almost unnecessary. But contributions continue to be added at a rate which soon makes the best summary incomplete, and, to a greater extent than I am aware, some of the results of this paper may be repetitions of work already published. Even then, however, I hope they may not be without value, on account of the facts (1) that, with the single exception of the experiment of § 121, the magnetisation dealt with has been as nearly uniform as it is practicable to have it, and (2) that very nearly all the observed values of magnetism and magnetising force have been reduced to absolute measure. It may be added that the subject with which this paper deals has, in addition to its scientific interest, a special practical importance at the present time, on account of its very obvious bearing on many points relating to the industrial applications of electricity, and particularly to the design of dynamo-electric machines.

§ 123. The work described in this paper was done during 1881–1883 in the Physical Laboratory of the University of Tokio, Japan, with the help of four Japanese students, without whose assistance, both in the taking of observations and in the arithmetical labour of reducing them, it would have been impossible for me to have covered so much ground. To Mr. R. Fujisawa and Mr. S. Tanaka I am particularly indebted for help in the first part of these experiments—the examination of the relation of $\mathfrak{I}$ to $\mathfrak{H}$ during the application and cyclic change of $\mathfrak{H}$, by the ballistic and magnetometric methods,—and to Mr. A. Tanakadate for help in the second part—the investigation of the effects of stress on magnetism. These gentlemen threw themselves into the work with a thoroughly intelligent interest and with the utmost zeal. In the last part of the paper—on the effects of temperature on magnetism—the observations which are quoted were for the most part taken under my direction by Mr. S. Sakai, who prosecuted this difficult portion of the subject with much perseverance, and to whom I am also indebted for valuable help in another investigation —of the effects of stress and magnetisation on the thermo-electric quality of iron—the results of which remain to be described. Those results, which are closely related to some parts of the foregoing inquiry, will form the subject of another paper.

Fig. 3.

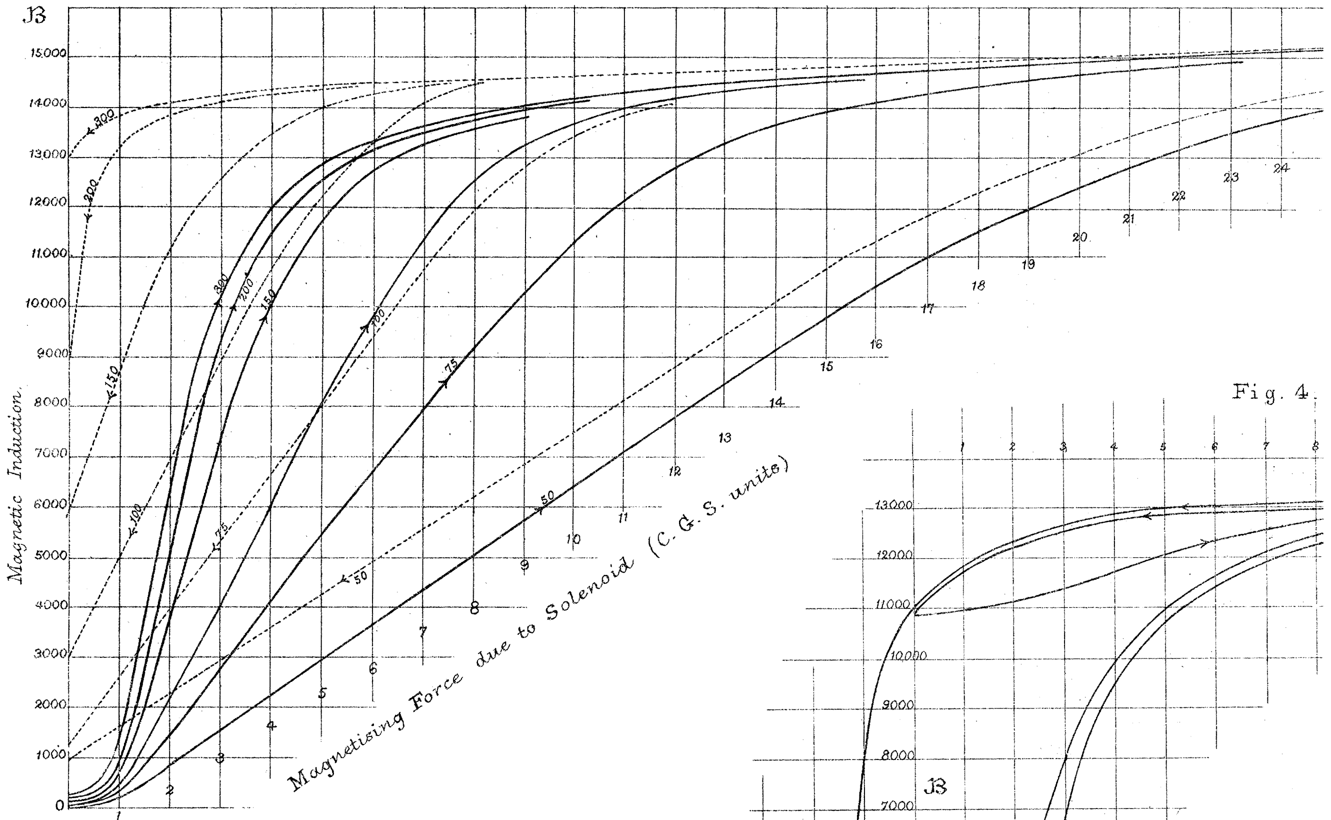
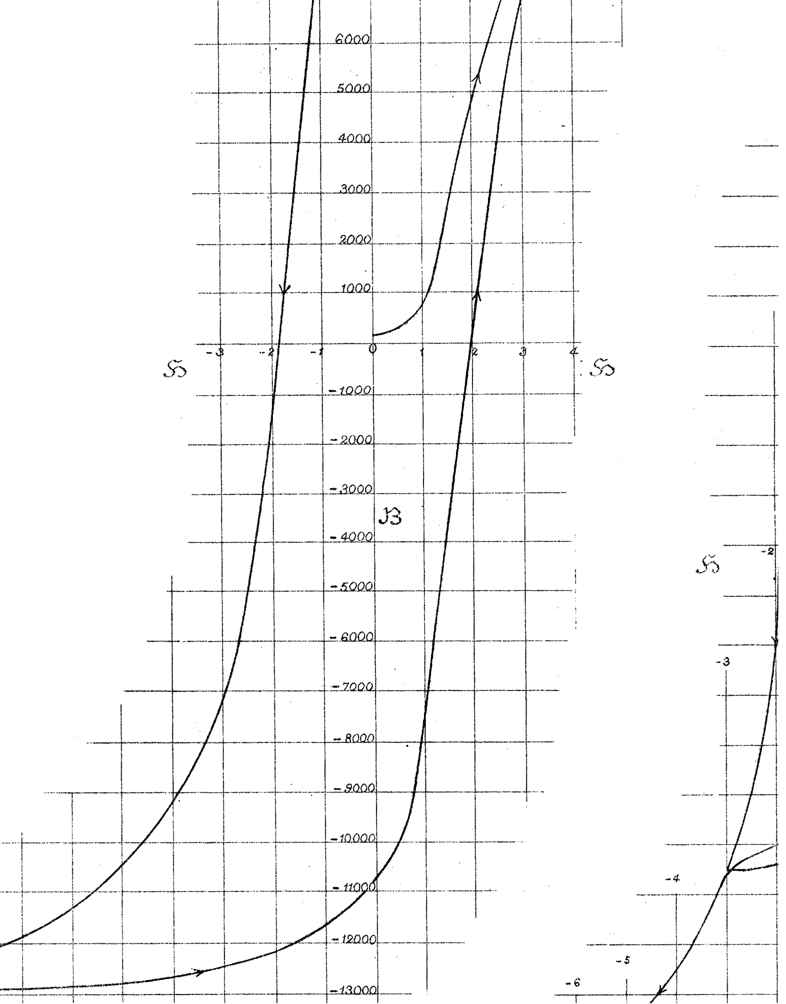


Fig. 4.



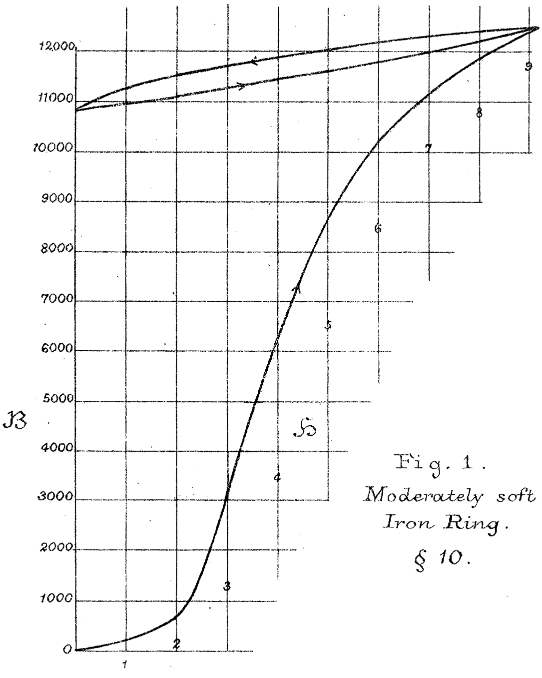
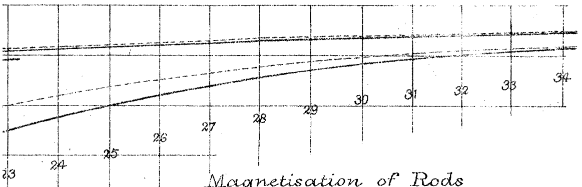
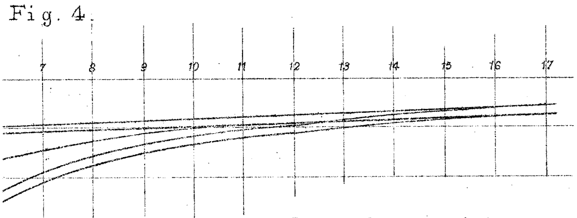


Fig. 1.
Moderately soft Iron Ring.
§ 10.



Magnetisation of Rods
of various Lengths.
The Numbers affixed to the Curves show the
ratio of length to diameter.—
§ 15.

Fig. 4.



Very soft annealed
Iron wire.
Length = 400 diameters. § 20.

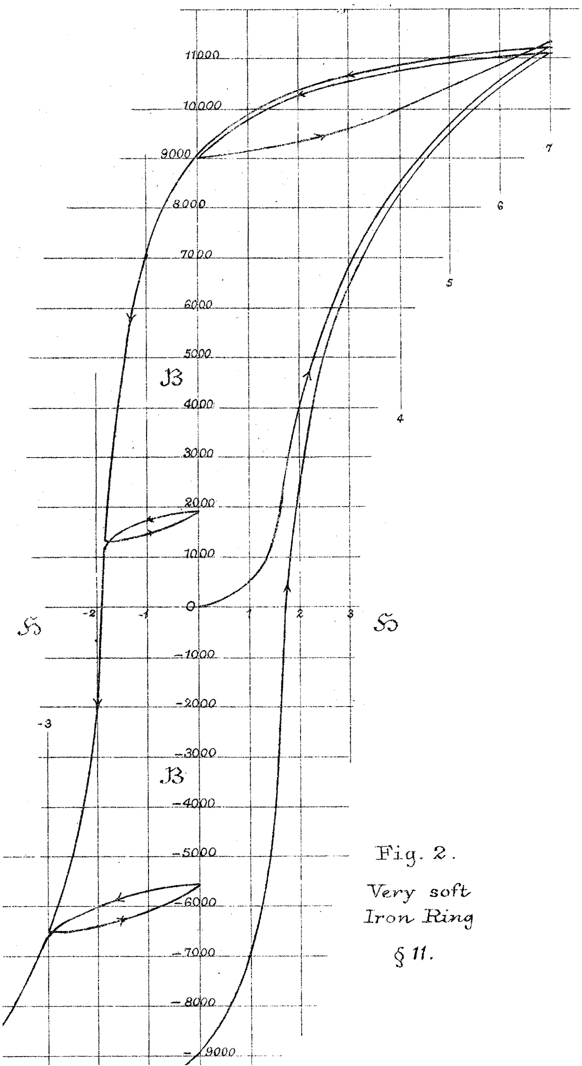
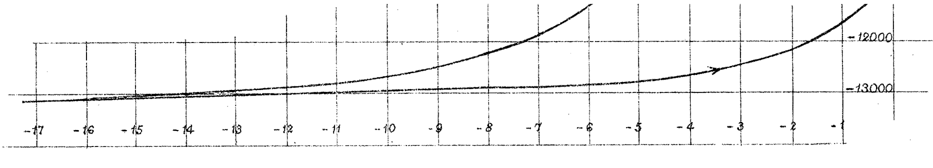


Fig. 2.

Very soft
Iron Ring

§ 11.



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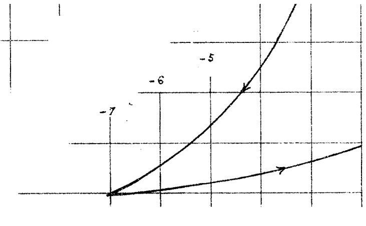

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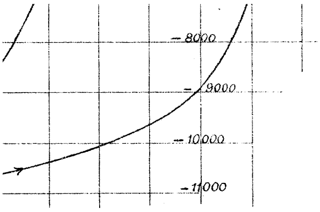

West, Newman & Cº lith.

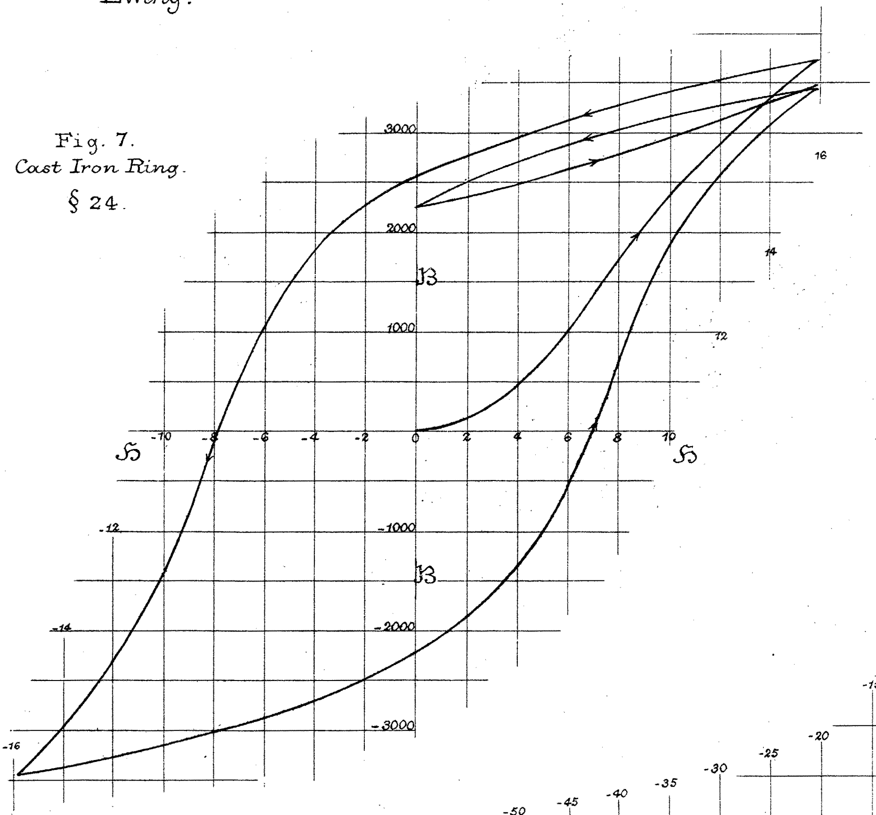

Fig. 7.
Cast Iron Ring.
§ 24.

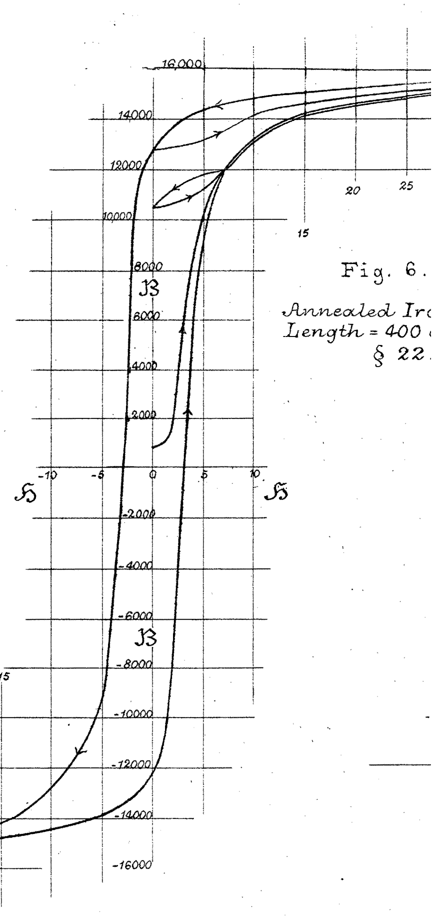

Fig. 6.
Annealed Ir
Length = 400
§ 22.

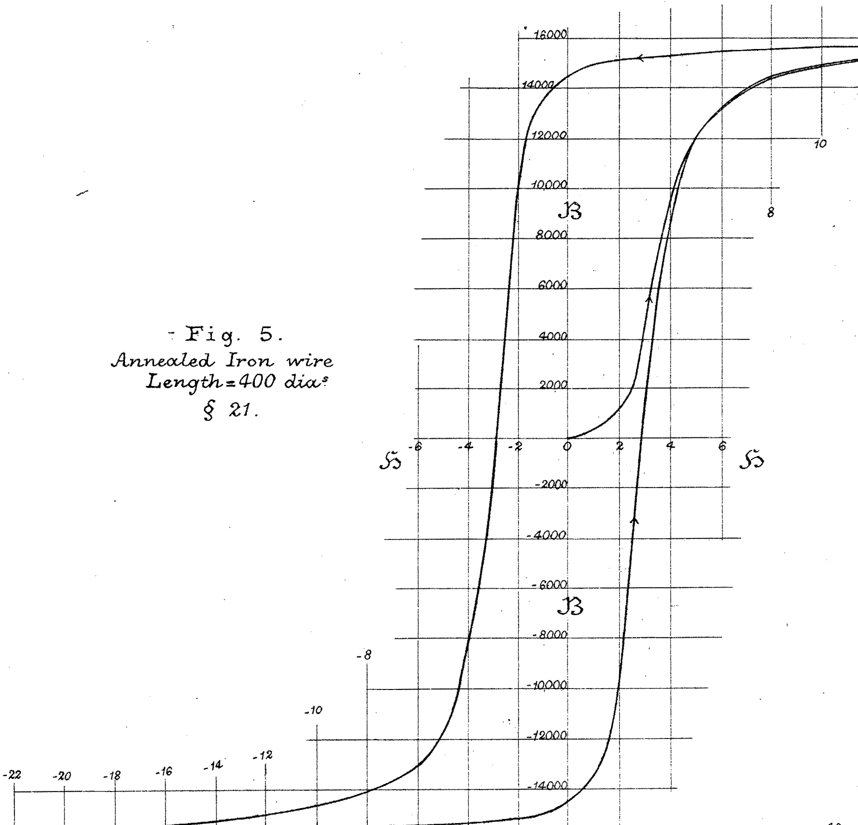

Fig. 5.
Annealed Iron wire
Length = 400 dia^s
§ 21.

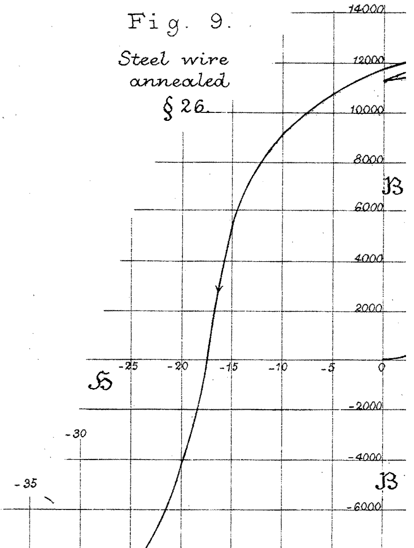

Fig. 9.
Steel wire
annealed
§ 26.

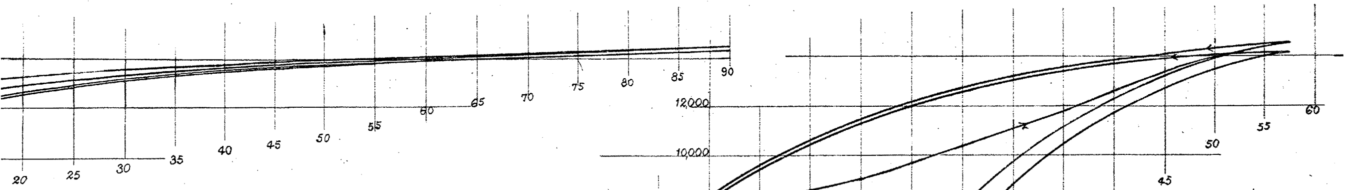

Fig. 6.

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th = 400 diam^s
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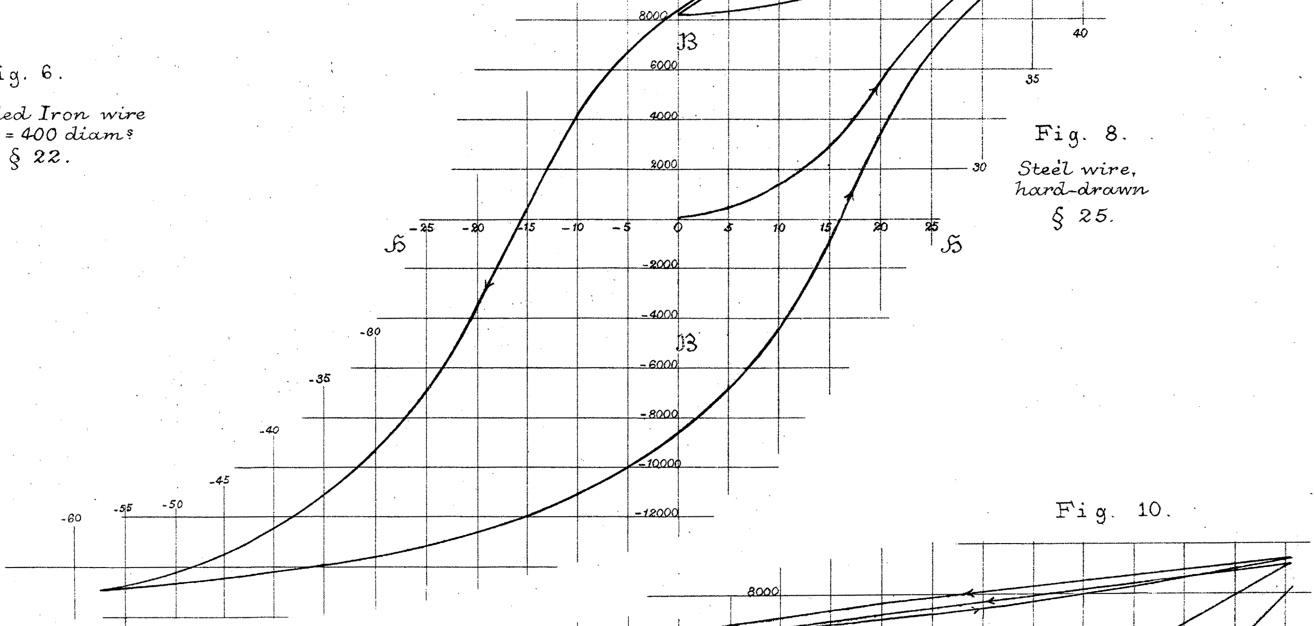
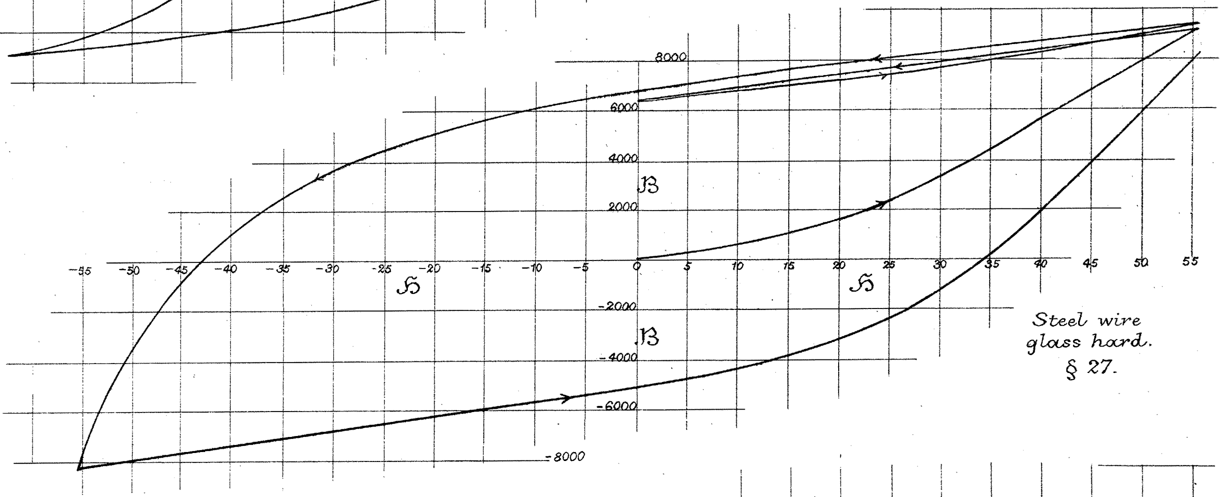


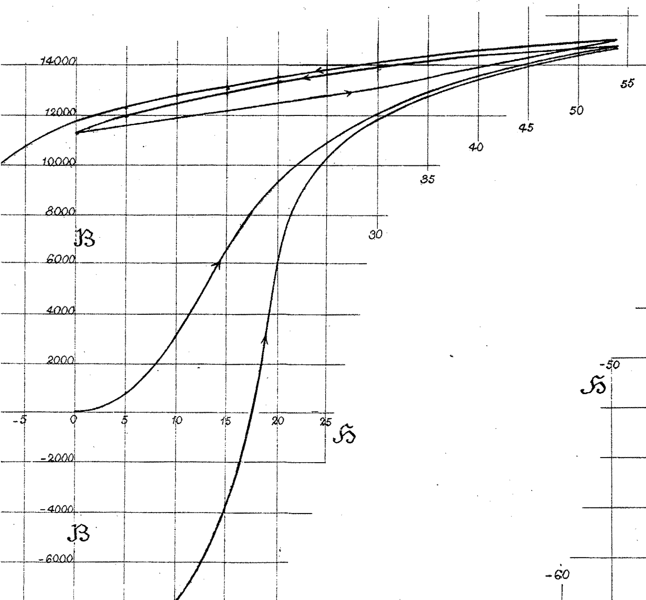
Fig. 8.

Steel wire,
hard-drawn
§ 25.

Fig. 10.



Steel wire
glass hard.
§ 27.



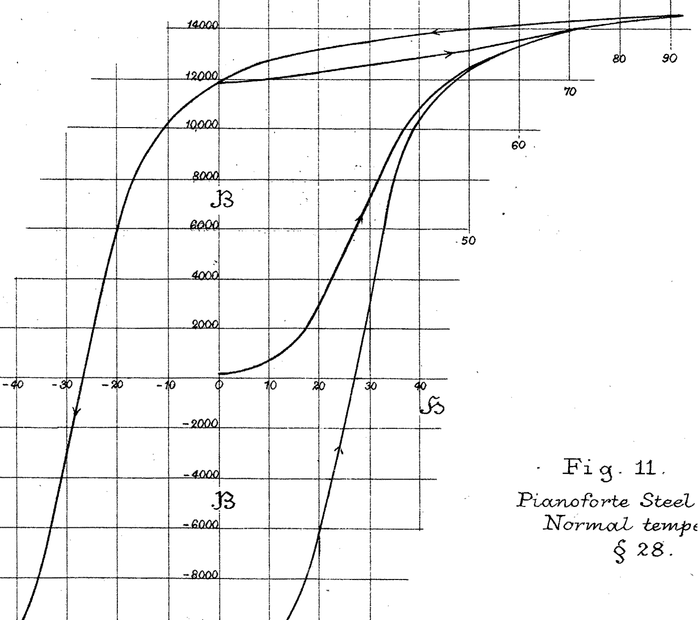


Fig. 11.

Pianoforte Steel
Normal tempe
§ 28.

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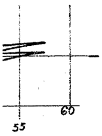

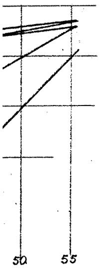

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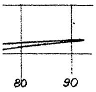

g. 11.
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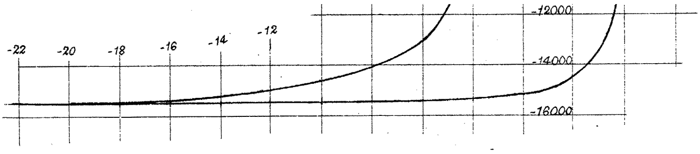
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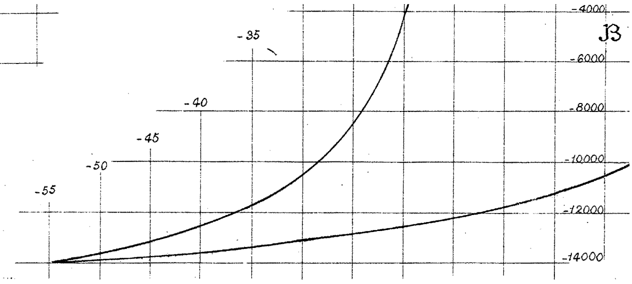
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Normal tempe

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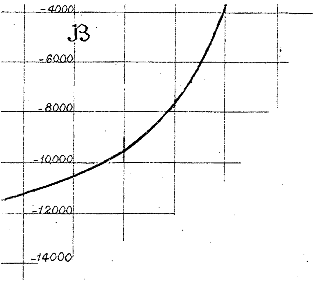
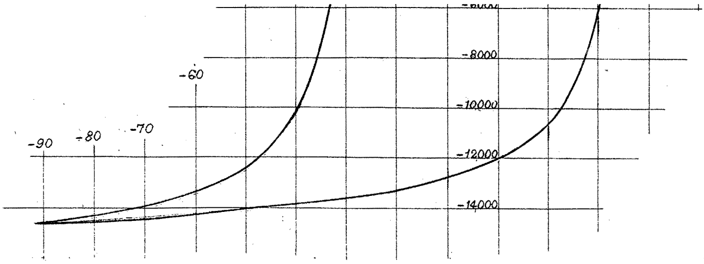


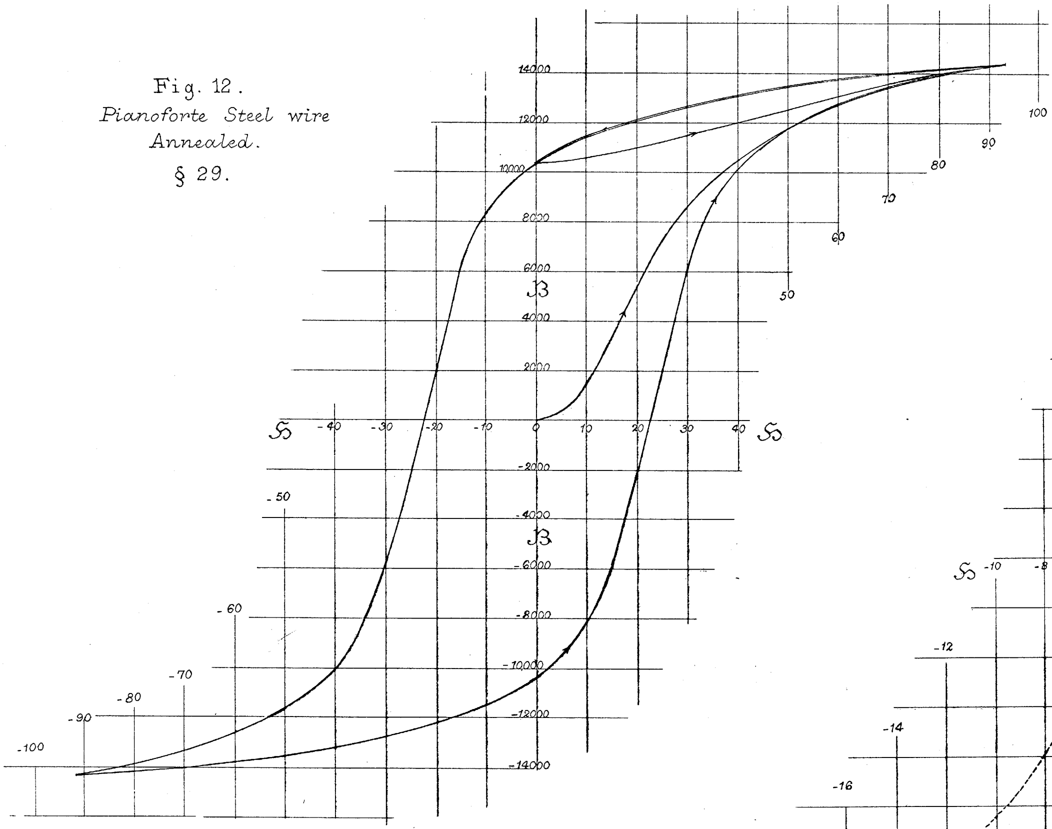


West, Newman & C° lith.

ai temper

§ 28.

Fig. 12.
Pianoforte Steel wire
Annealed.
§ 29.



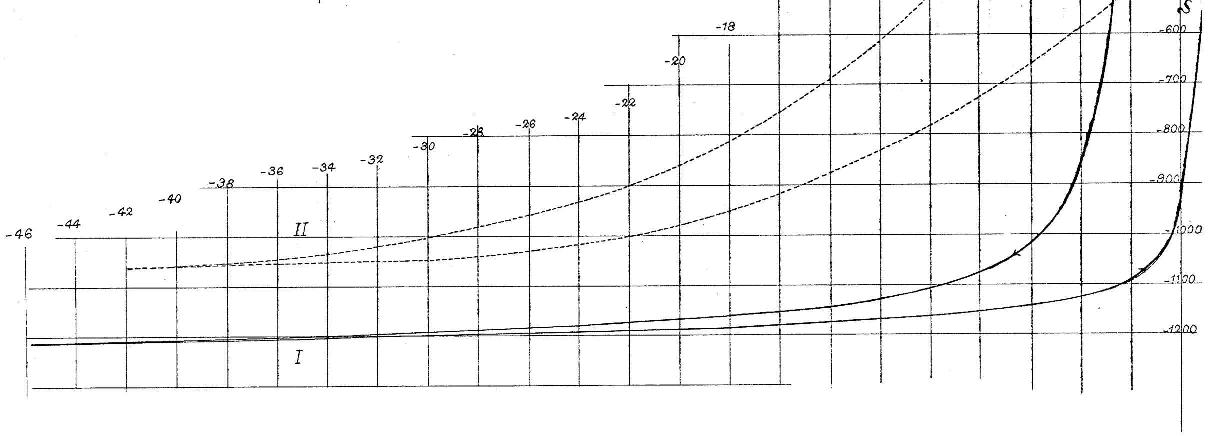
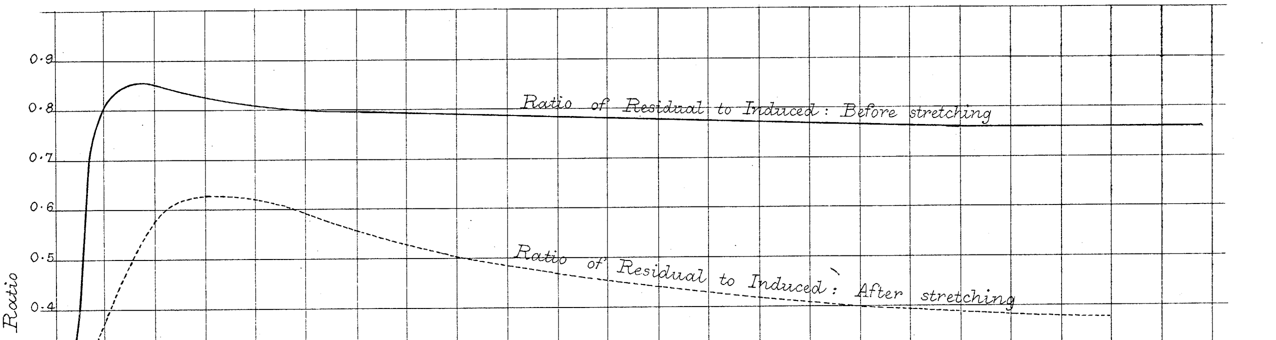


Fig. 18.



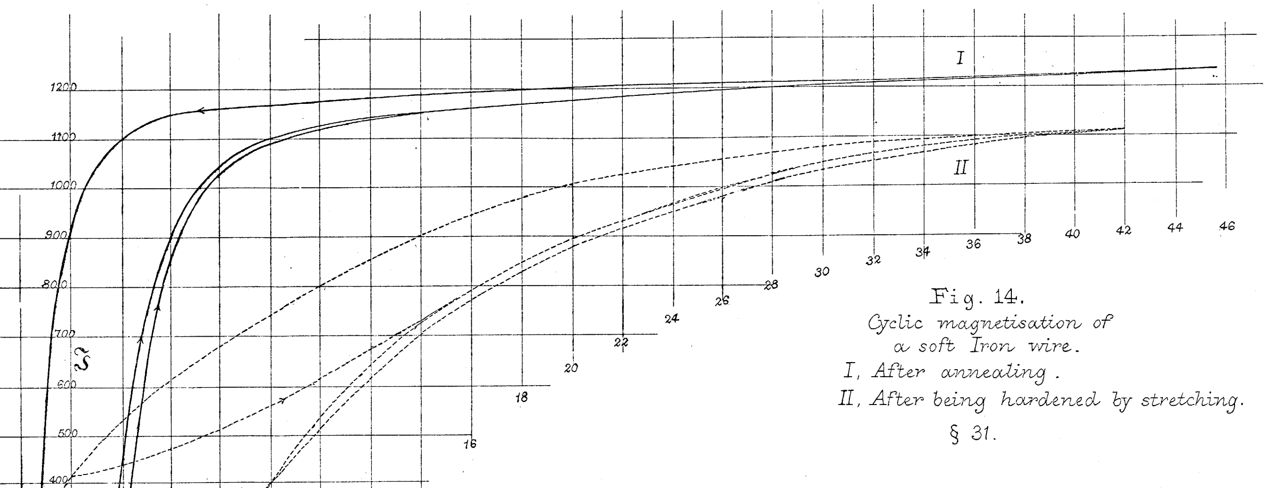


Fig. 14.
Cyclic magnetisation of a soft Iron wire.
I, After annealing.
II, After being hardened by stretching.
§ 31.

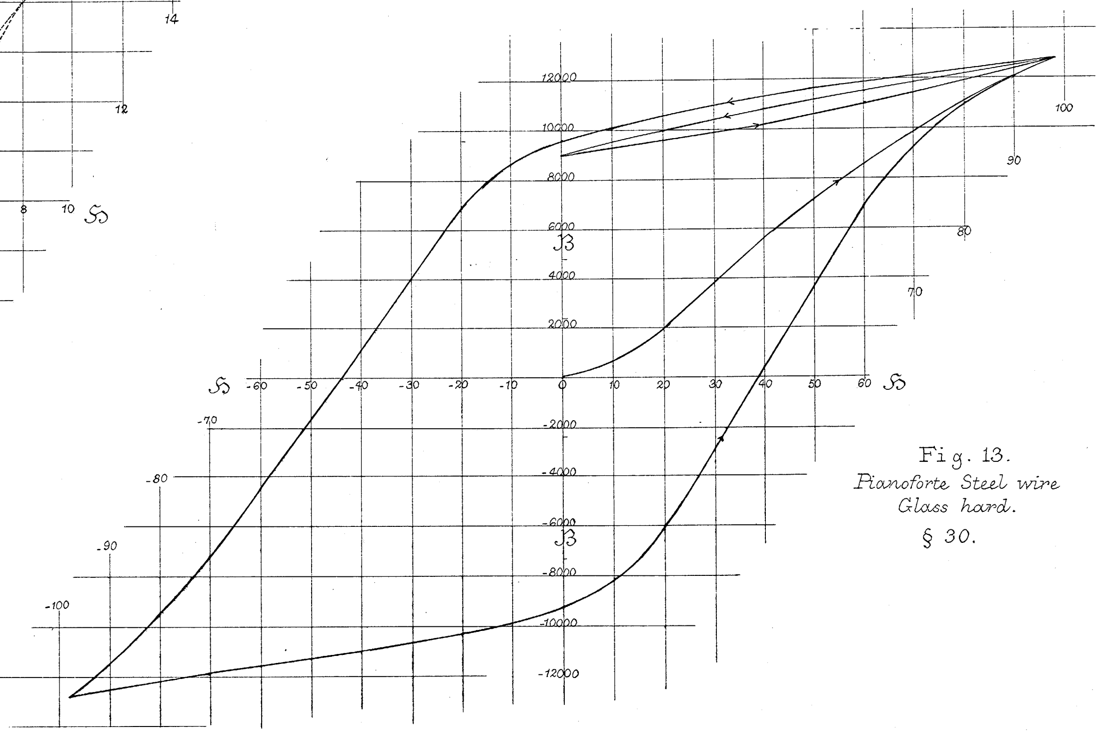


Fig. 13.
Pianoforte Steel wire
Glass hard.
§ 30.

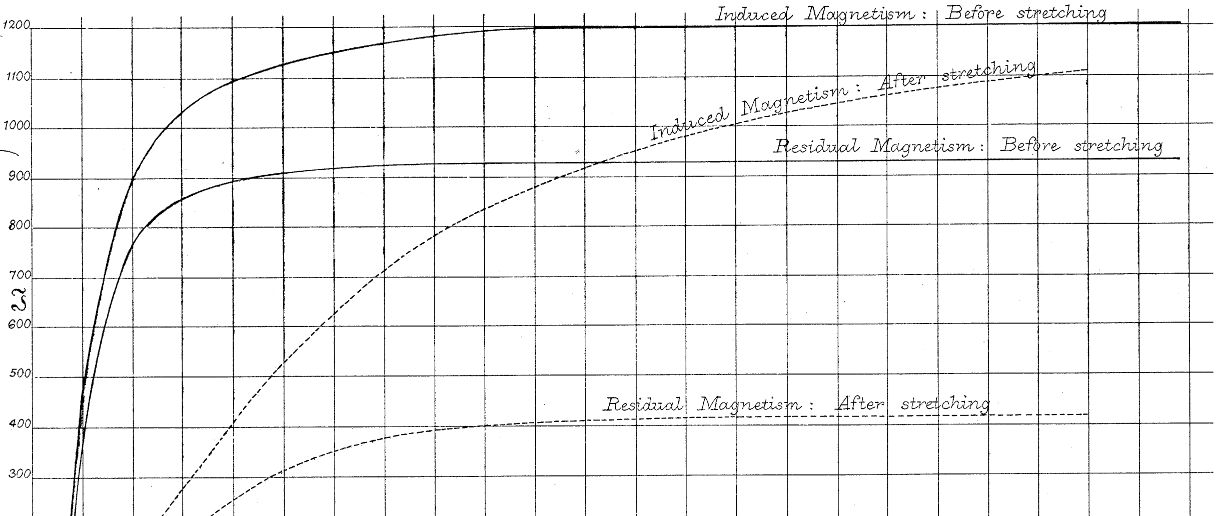
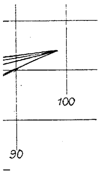


Fig. 18.



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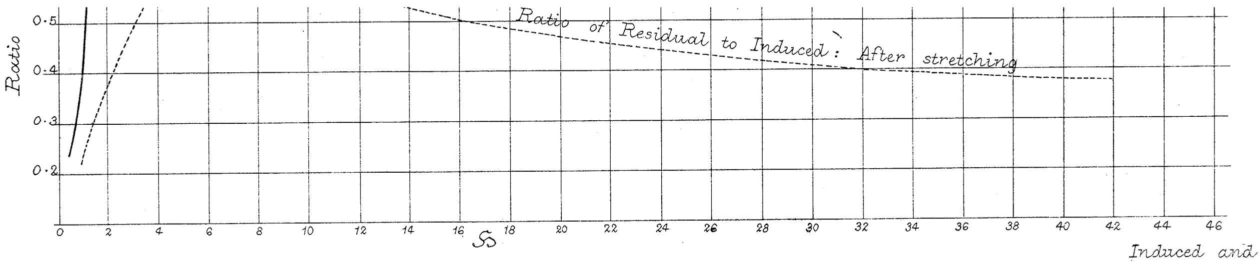

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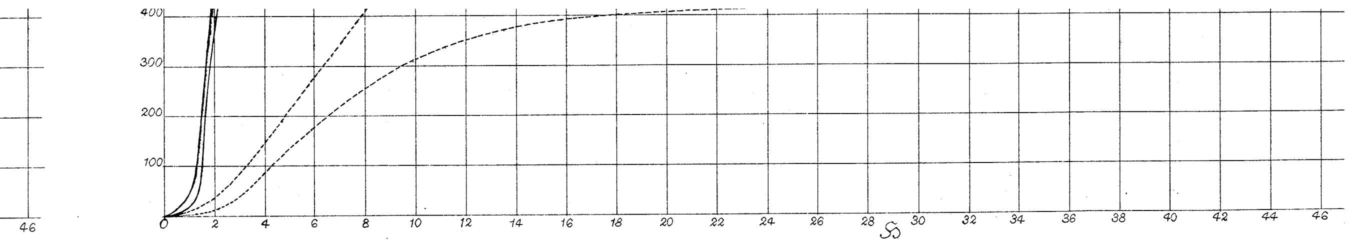
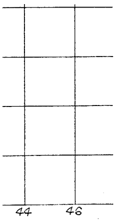


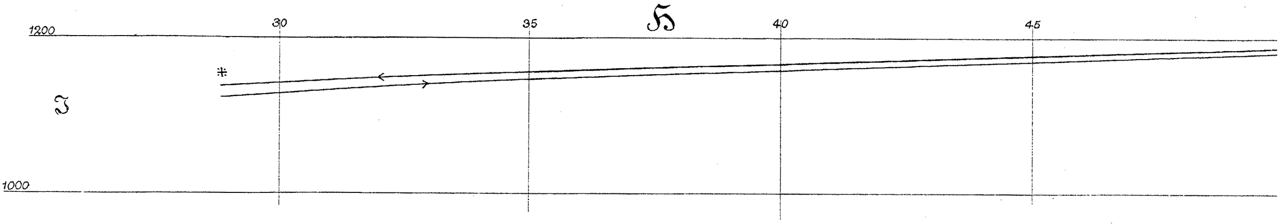
ed and Residual Magnetism in Iron.
Before stretching ————
After stretching ------------

§ 41.

West Newman & Co. lith.



& C° lith.



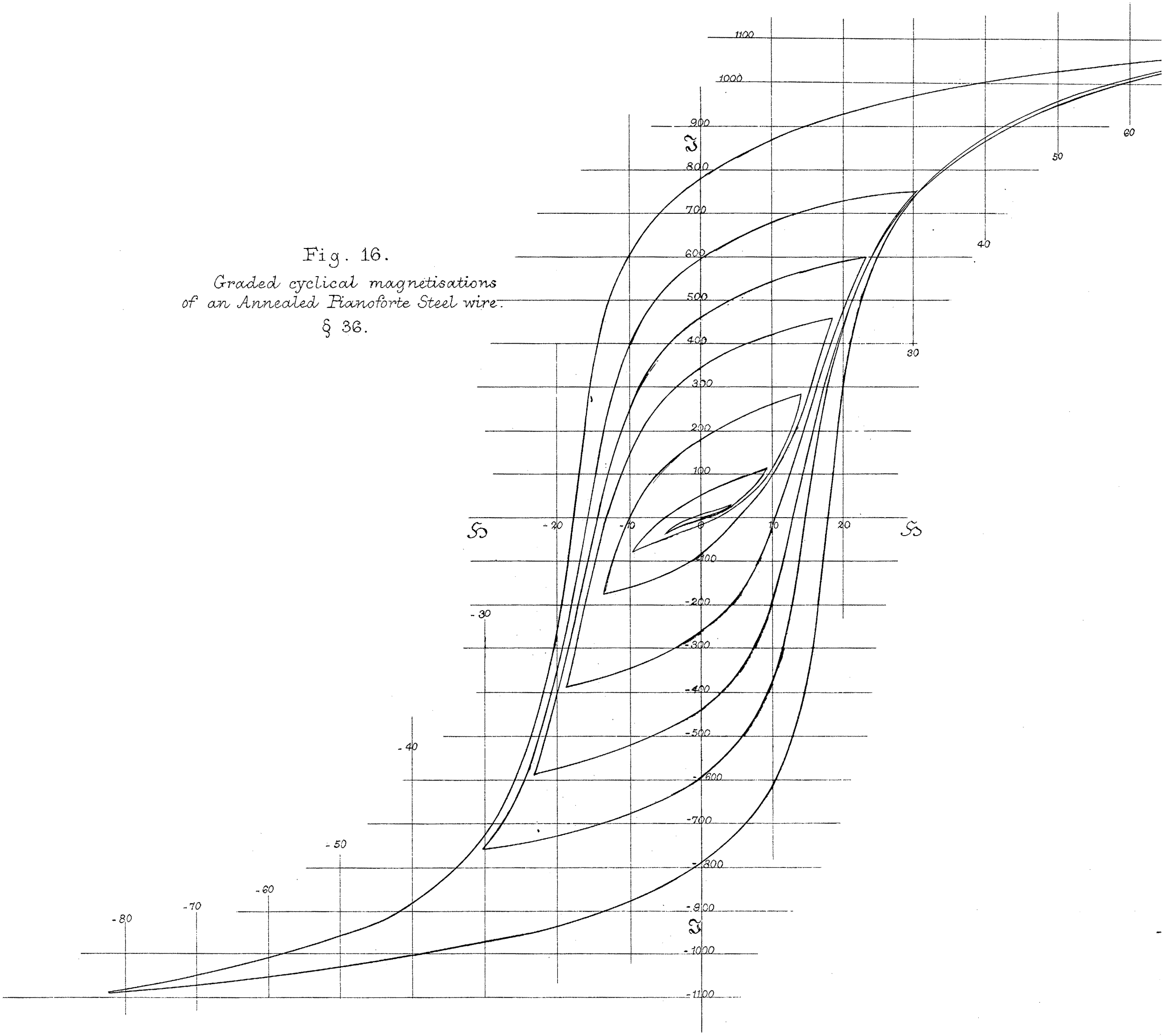
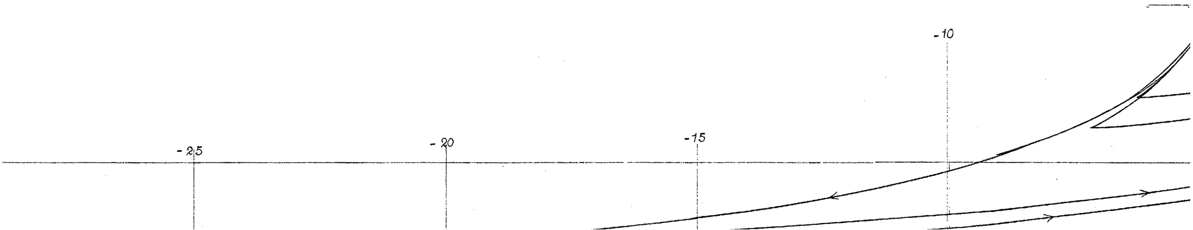


Fig. 16.

Graded cyclical magnetisations of an Annealed Pianoforte Steel wire.

§ 36.



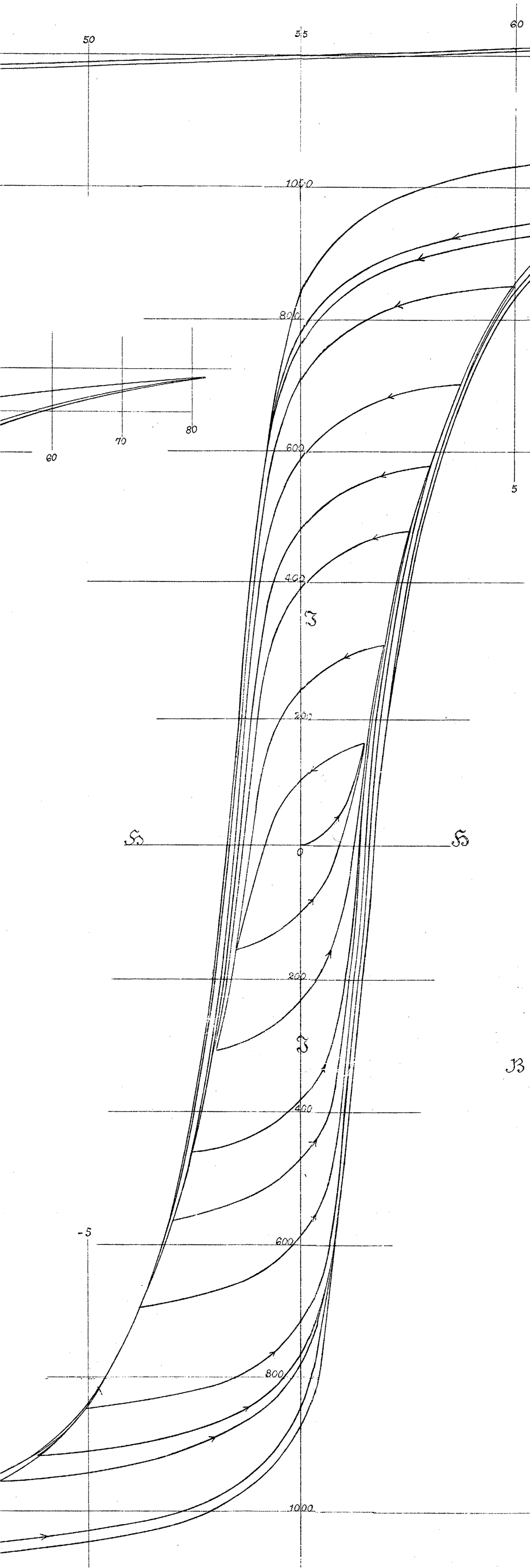


Fig. 15.

Graded cyclical magnetisations of a soft Iron wire.

§ 35.

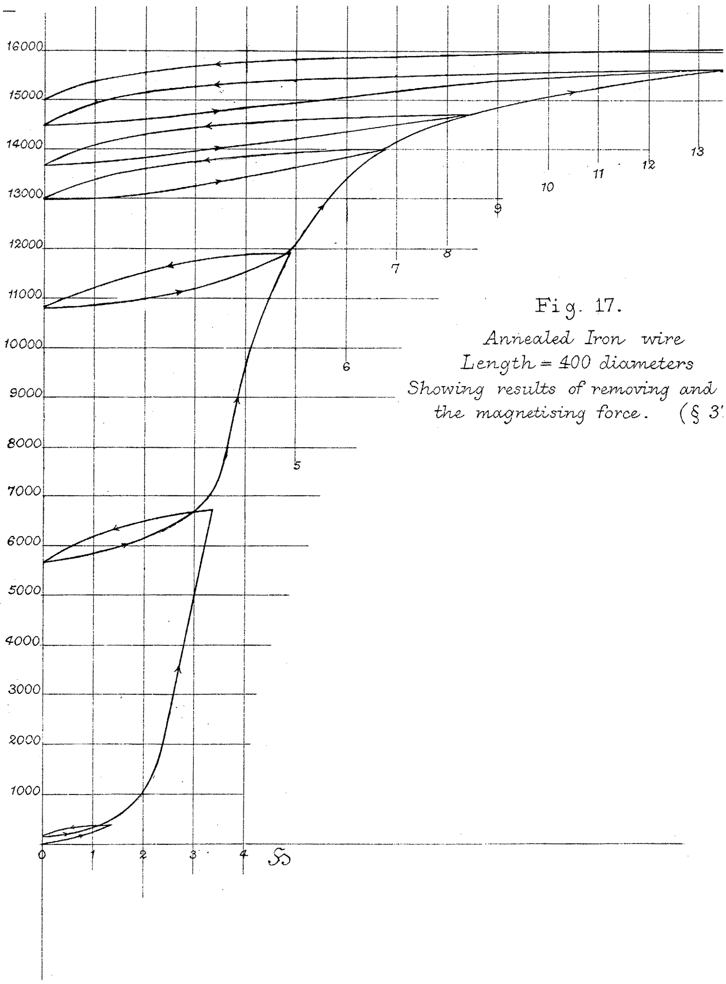


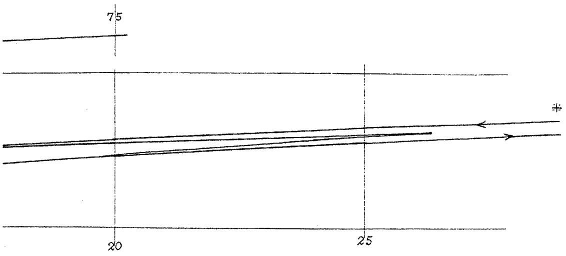
Fig. 17.

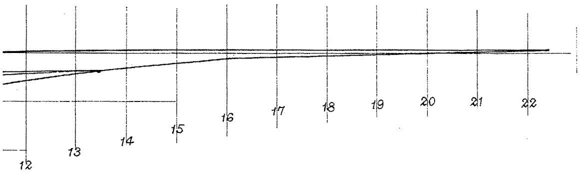
Annealed Iron wire

Length = 400 diameters

Showing results of removing and

the magnetising force. (§ 3





wire
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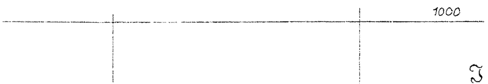

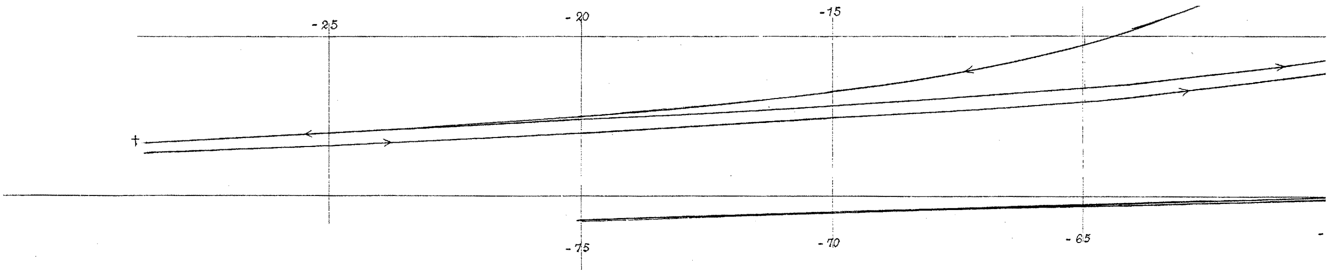
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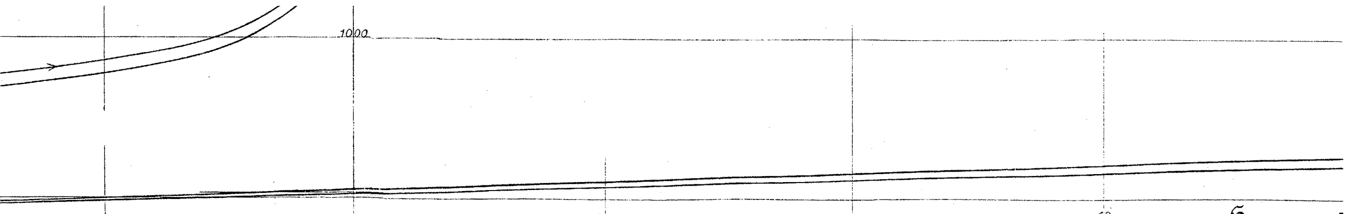
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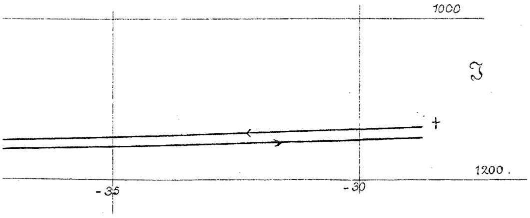


West Newman & C° lith.

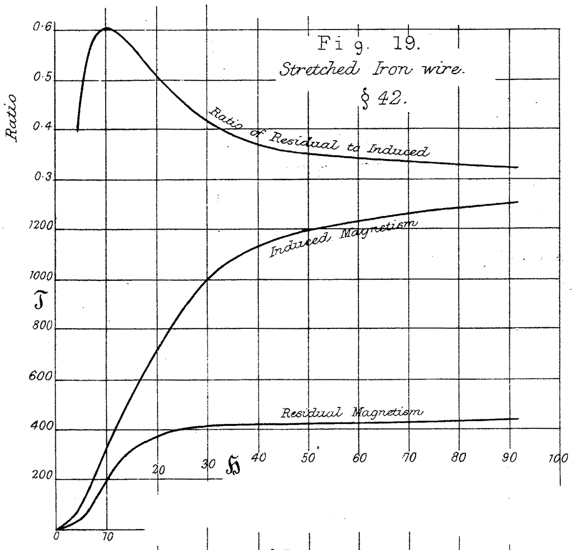


Fig. 19.
Stretched Iron wire.
§ 42.

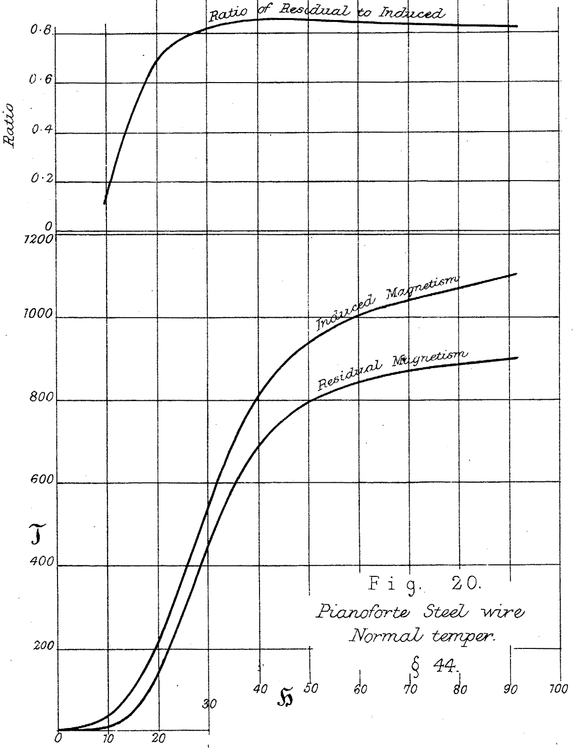
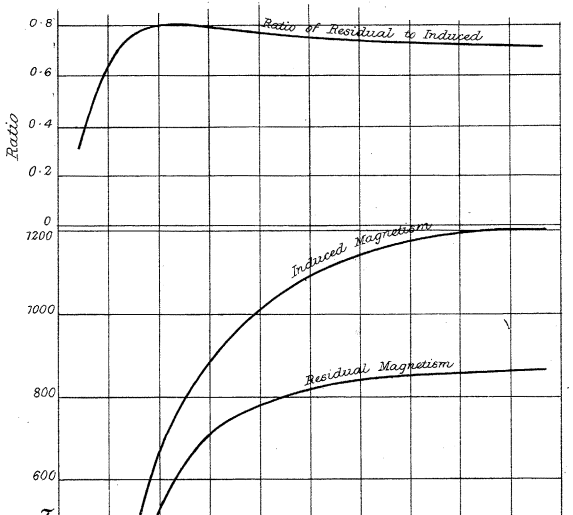


Fig. 20.
Pianoforte Steel wire
Normal temper.
§ 44.



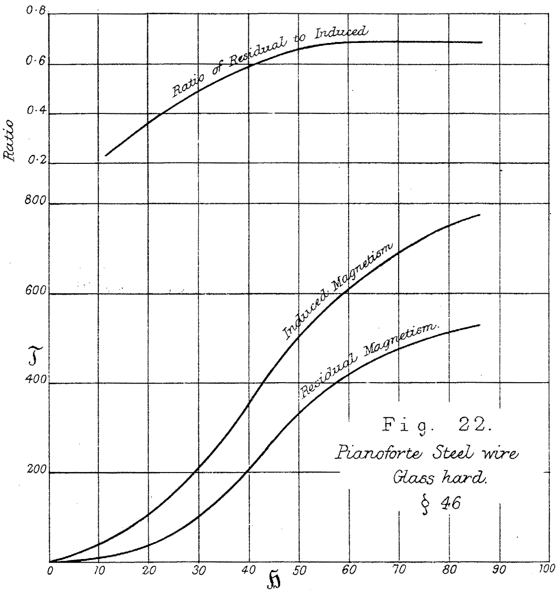
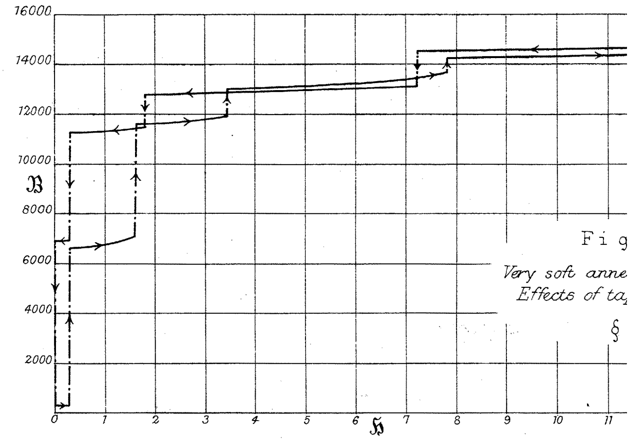


Fig. 22.
Pianoforte Steel wire
Glass hard.
§ 46



Fig
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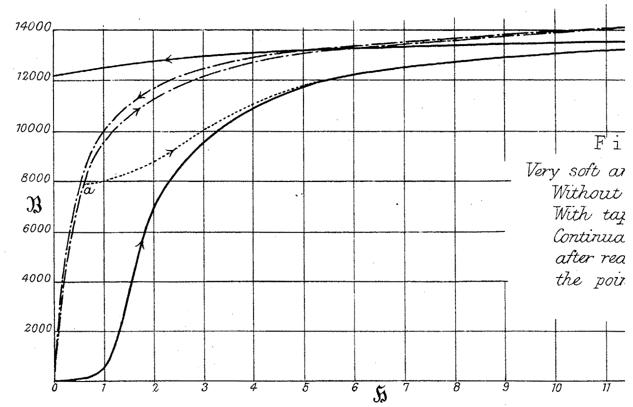
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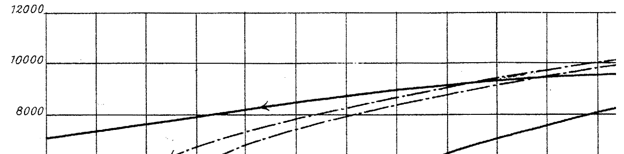


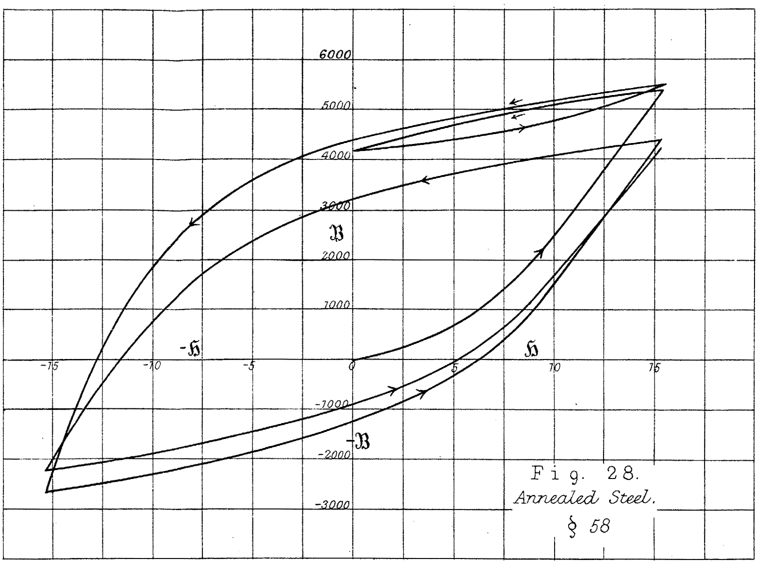


Fig. 28.
Annealed Steel.
§ 58

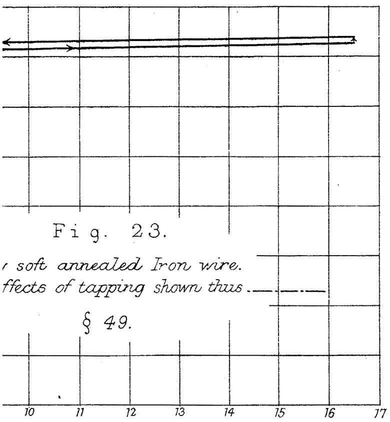


Fig. 23.
/ soft annealed Iron wire.
ffects of tapping shown thus ——·——
§ 49.

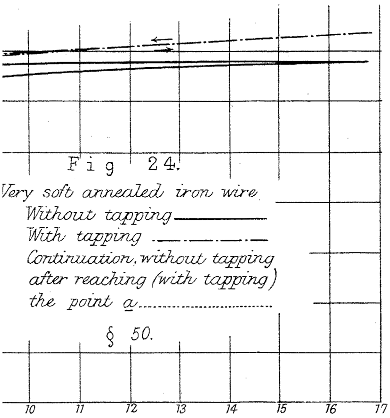
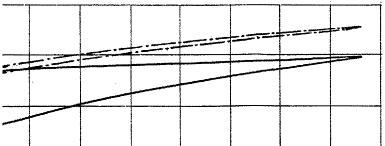


Fig 24.
Very soft annealed iron wire
Without tapping ————
With tapping ——·——·——
Continuation, without tapping
after reaching (with tapping)
the point a ....
§ 50.



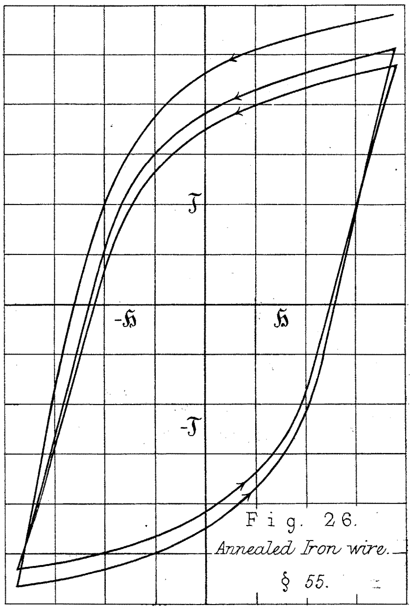
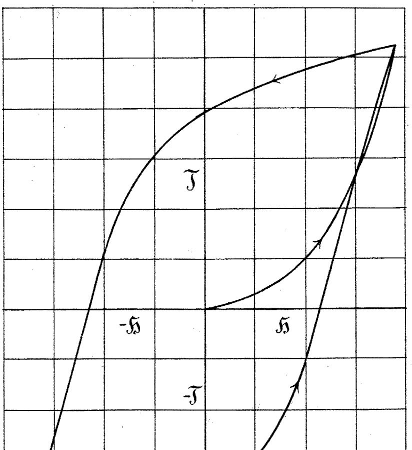


Fig. 26.
Annealed Iron wire.
§ 55.



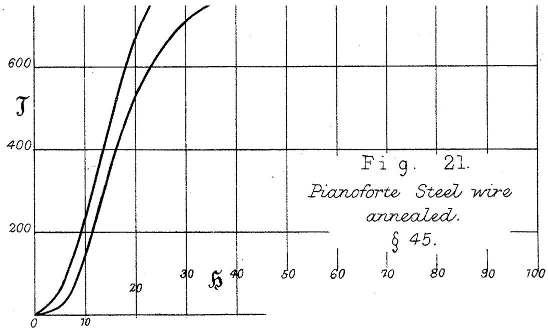
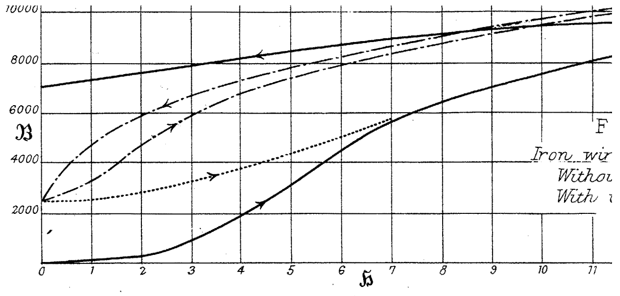


Fig. 21.

Pianoforte Steel wire annealed.

§ 45.



F

Iron wir

Withou

With

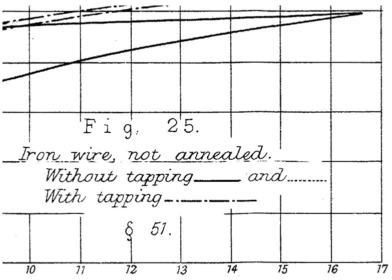


Fig. 25.

Iron wire, not annealed.
Without tapping ——— *and* ....
With tapping —·—·—

§ 51.

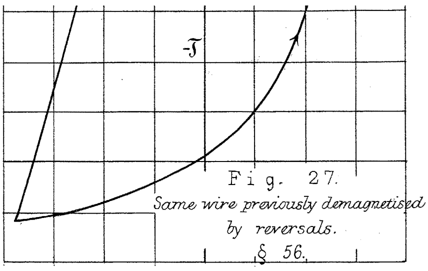


Fig. 27.

Same wire previously demagnetised by reversals.

§ 56.

West Newman & C^o. lith.

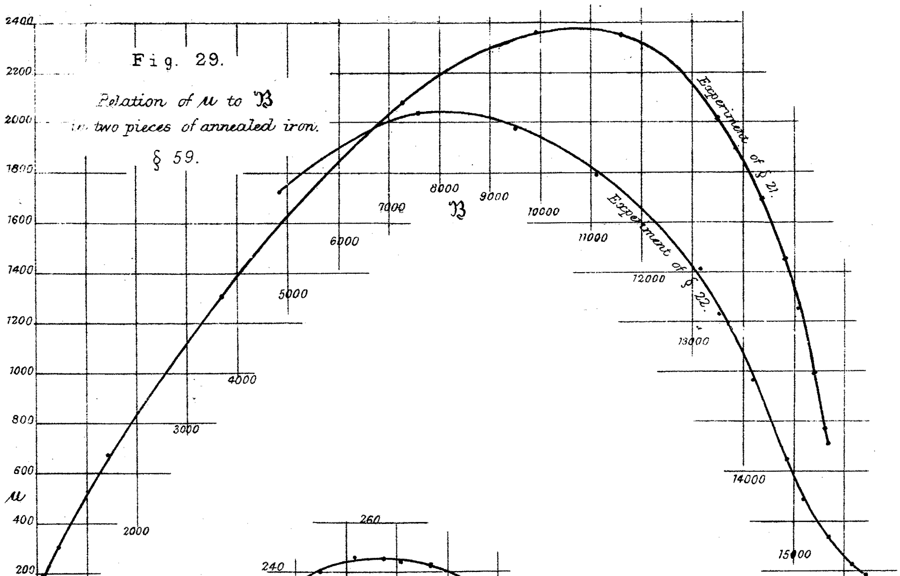


Fig. 29.

Relation of μ to $\mathfrak{B}$ in two pieces of annealed iron.

§ 59.

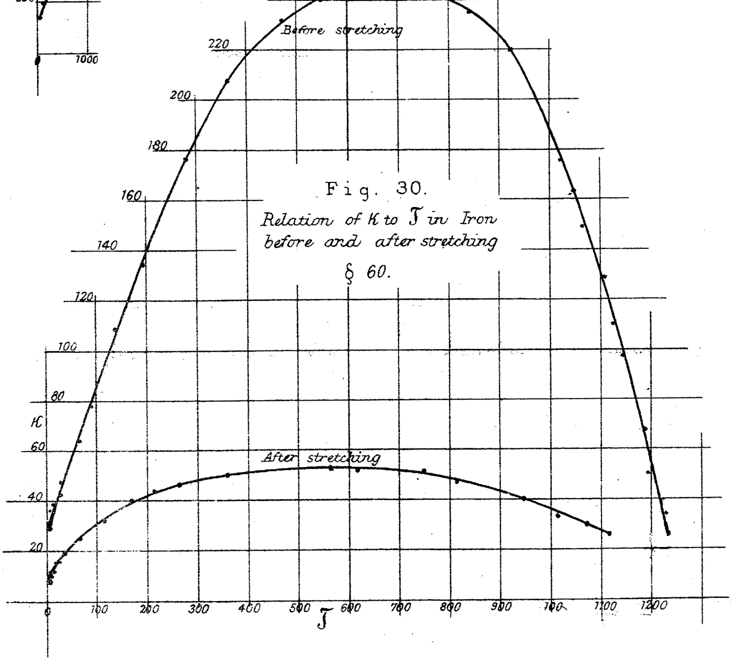


Fig. 30.

Relation of K to $\mathfrak{I}$ in Iron before and after stretching

§ 60.

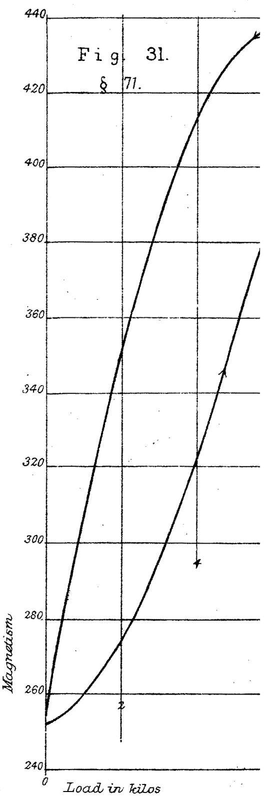


Fig. 31.

§ 71.

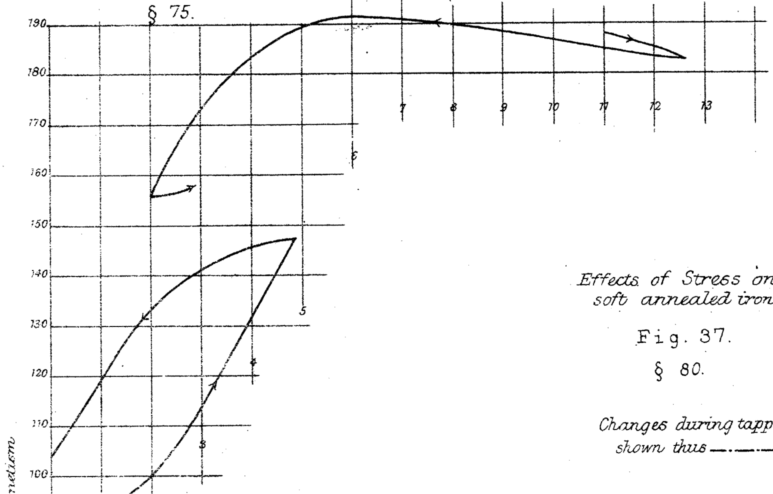
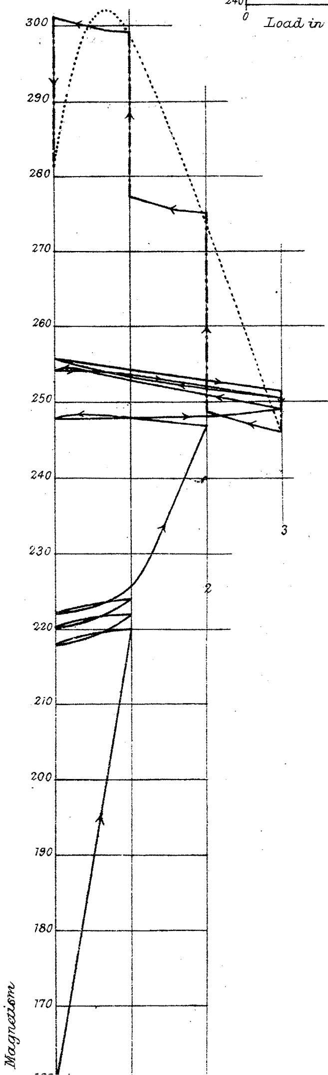


Fig. 35.

§ 75.



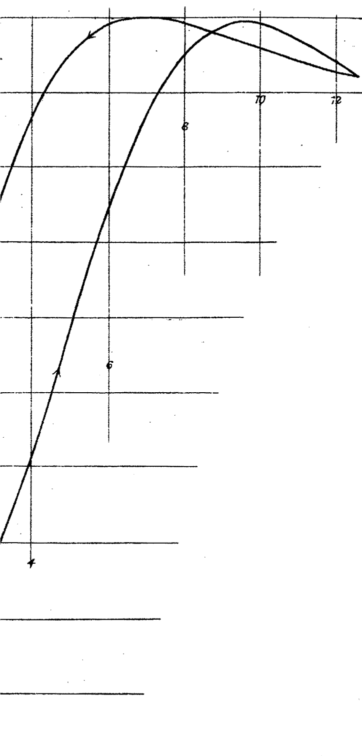
Effects of Stress on soft annealed iron.

Fig. 37.

§ 80.

Changes during tapping shown thus —·—·—

Effects of Stress on a Stretched Iron Wire in a Constant Magnetic Field (0·34 c.g.s. ur



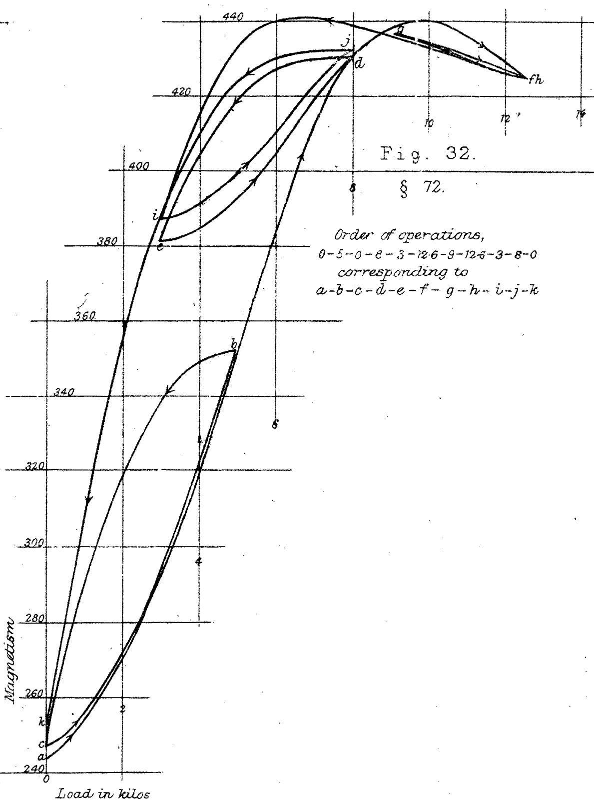


Fig. 32.
§ 72.

Order of operations,
0-5-0-e-3-12·6-9-12·6-3-8-0
corresponding to
a-b-c-d-e-f-g-h-i-j-k

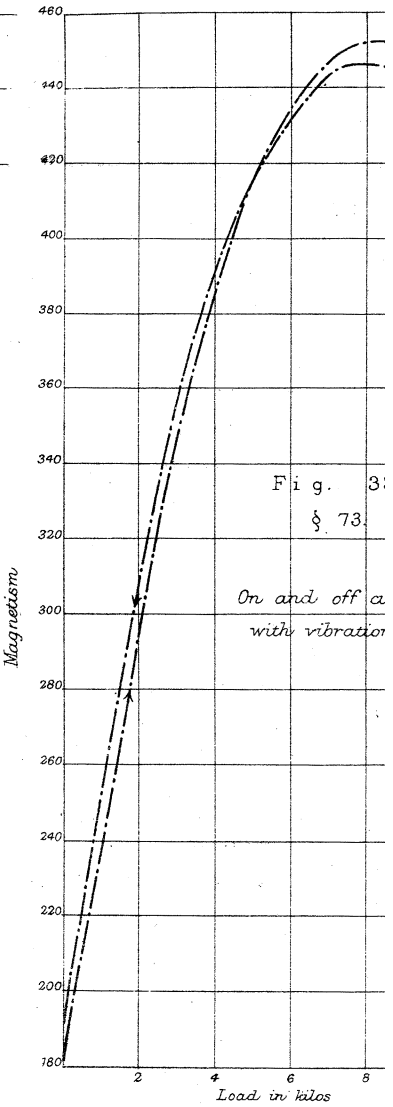
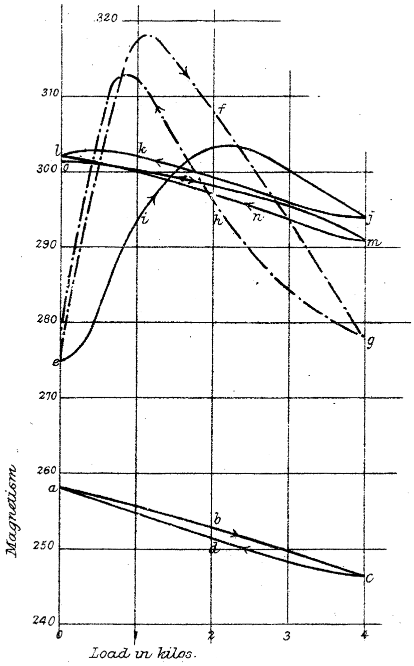


Fig. 3
§ 73.

On and off a
with vibration

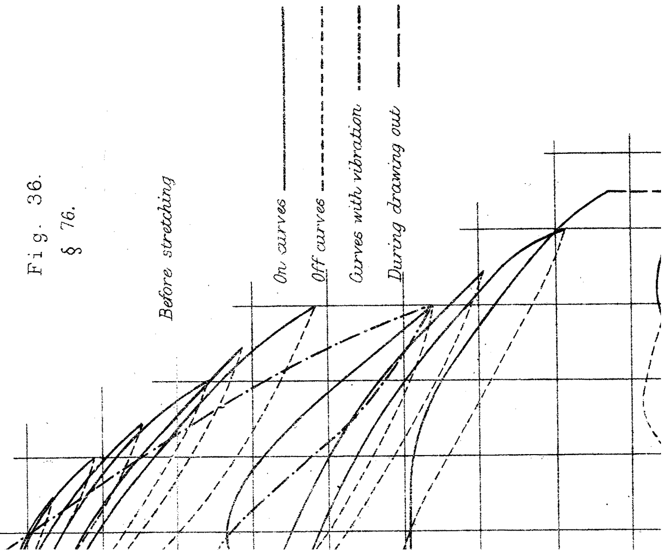
Fig. 38.
§ 82.



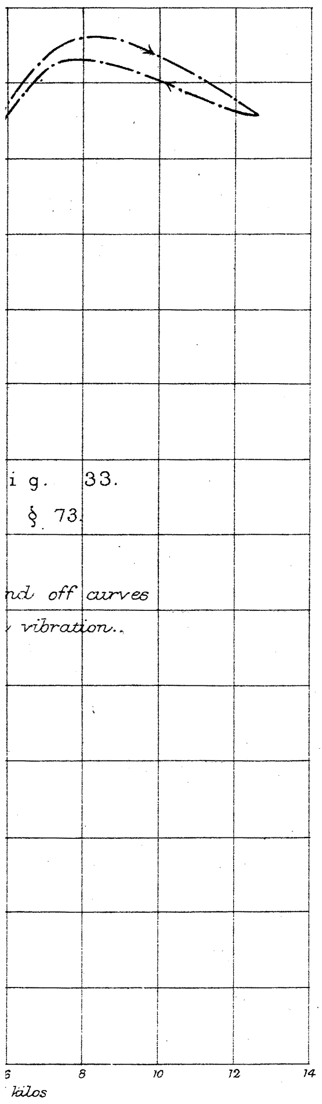
Effects of Stress on the Magnetism of Iron
in a constant field.

Fig. 36.
§ 76.

Before stretching



4 c.g.s. units).


ig. 33.

§ 73.

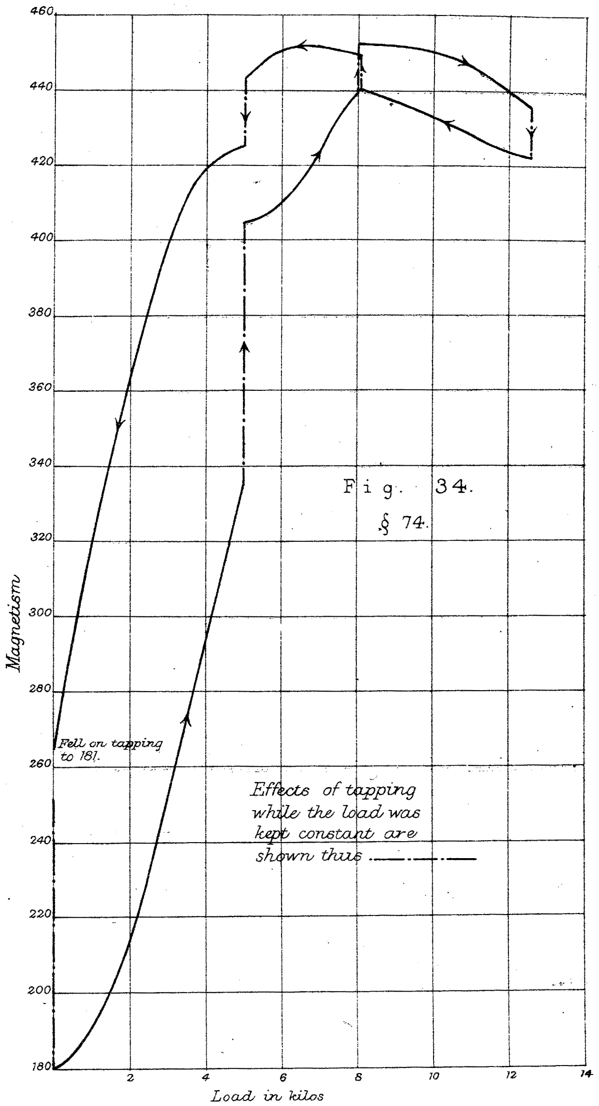
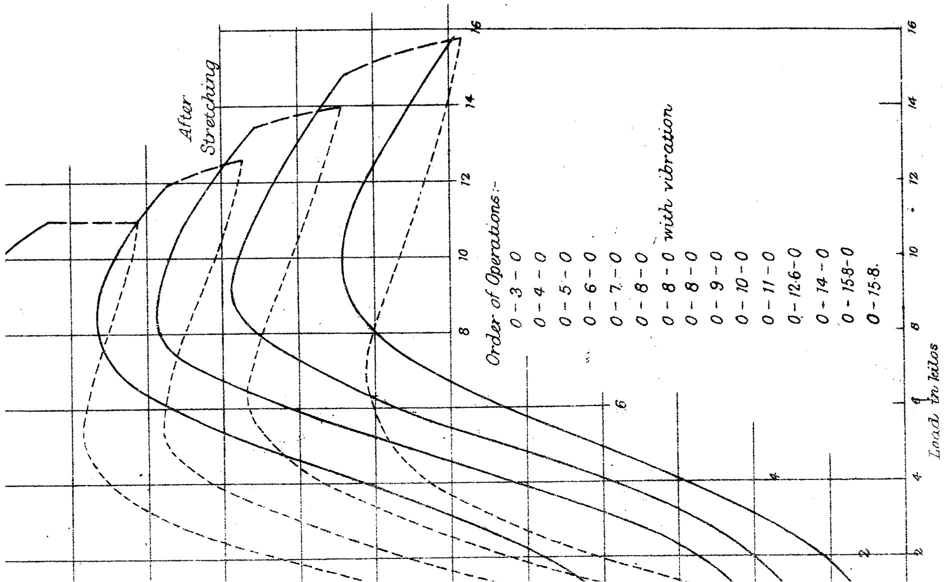
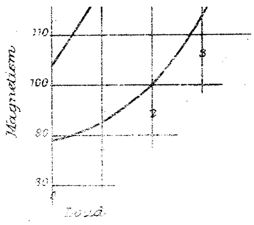


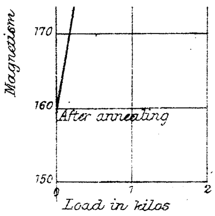
Fig. 34.

§ 74.

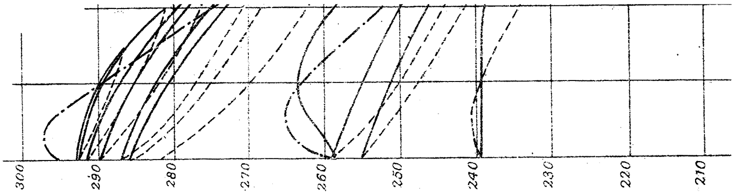


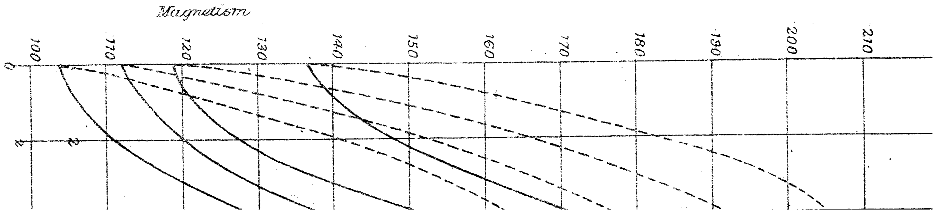


Changes during tapping
shown thus —·—·—



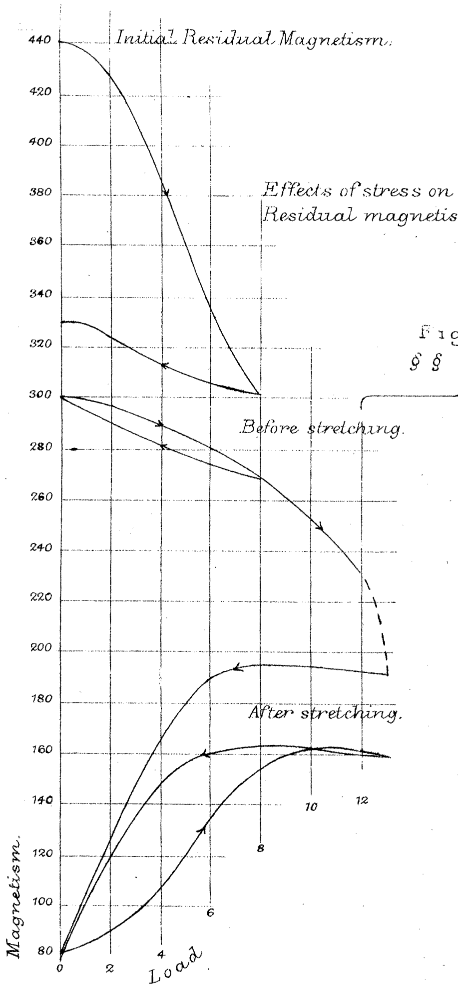
Load in kilos.

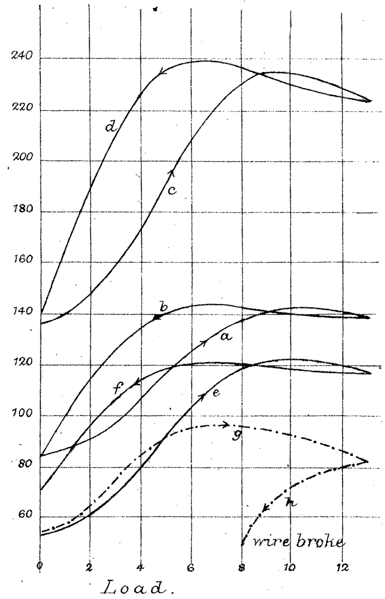




West Newman & Co. lith

Fig. 39.
§§ 83–85





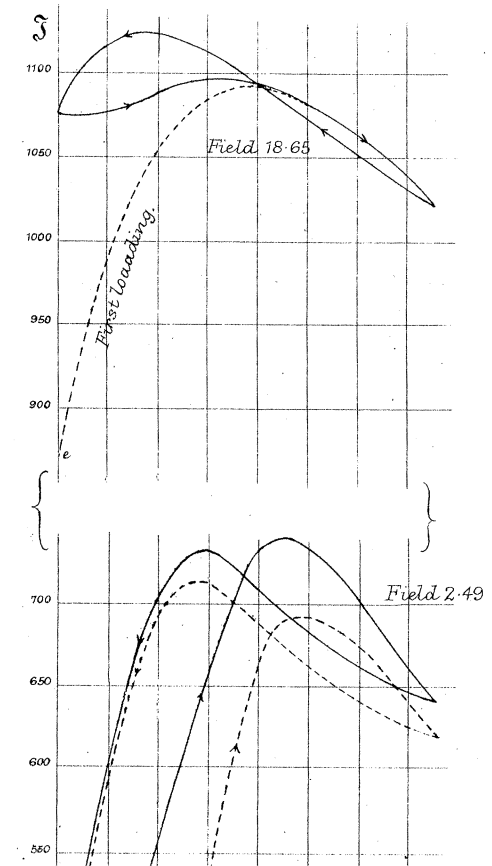
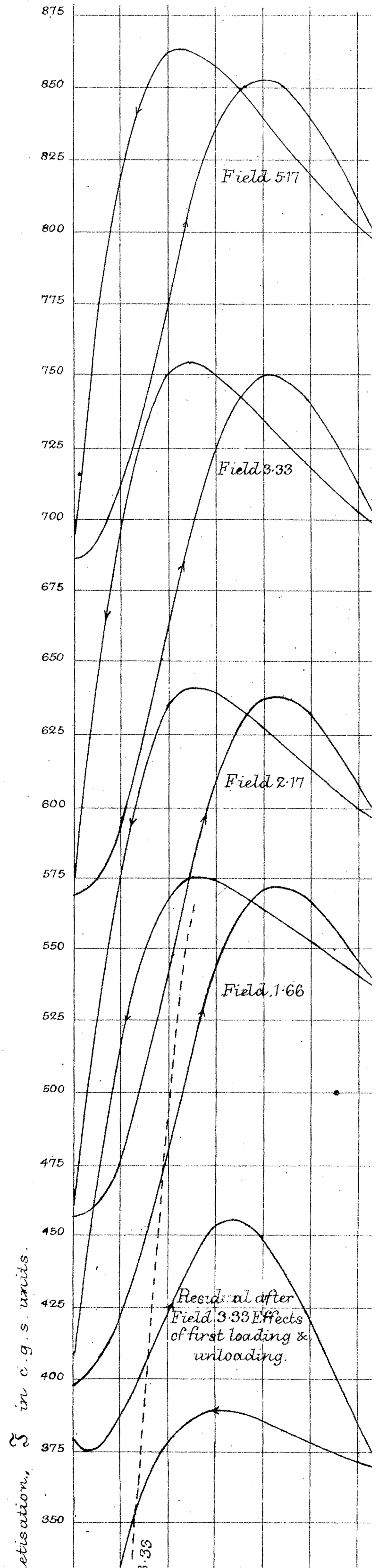


Fig. 40.
§ 86

Effects of stress on the Magnetism of a stretched iron wire in various fields.



the
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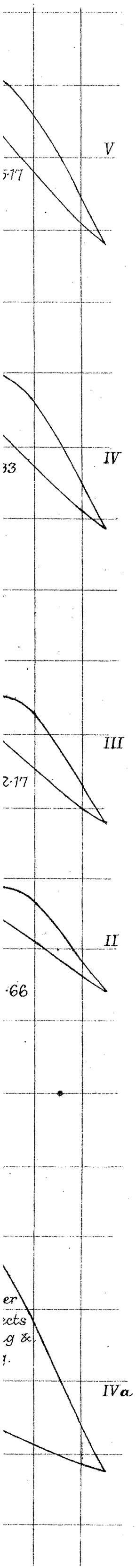


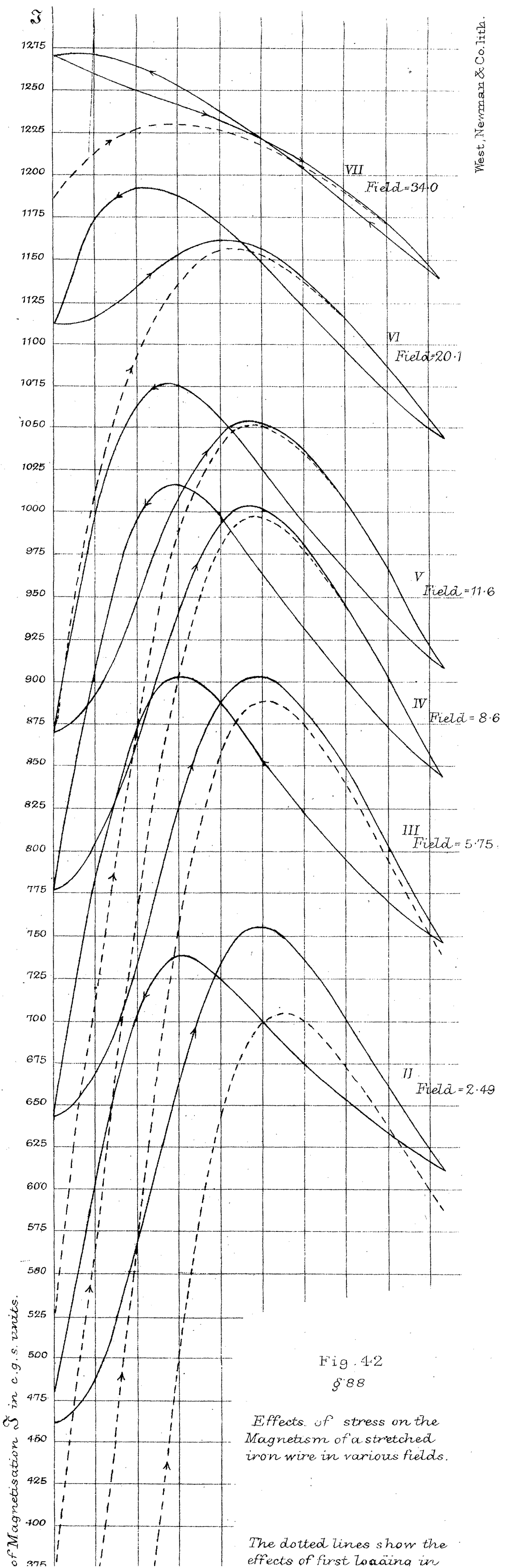


Fig. 42
§ 88

Effects of stress on the Magnetism of a stretched iron wire in various fields.

The dotted lines show the effects of first loading in

West, Newman & Co. lith.

Ewing.

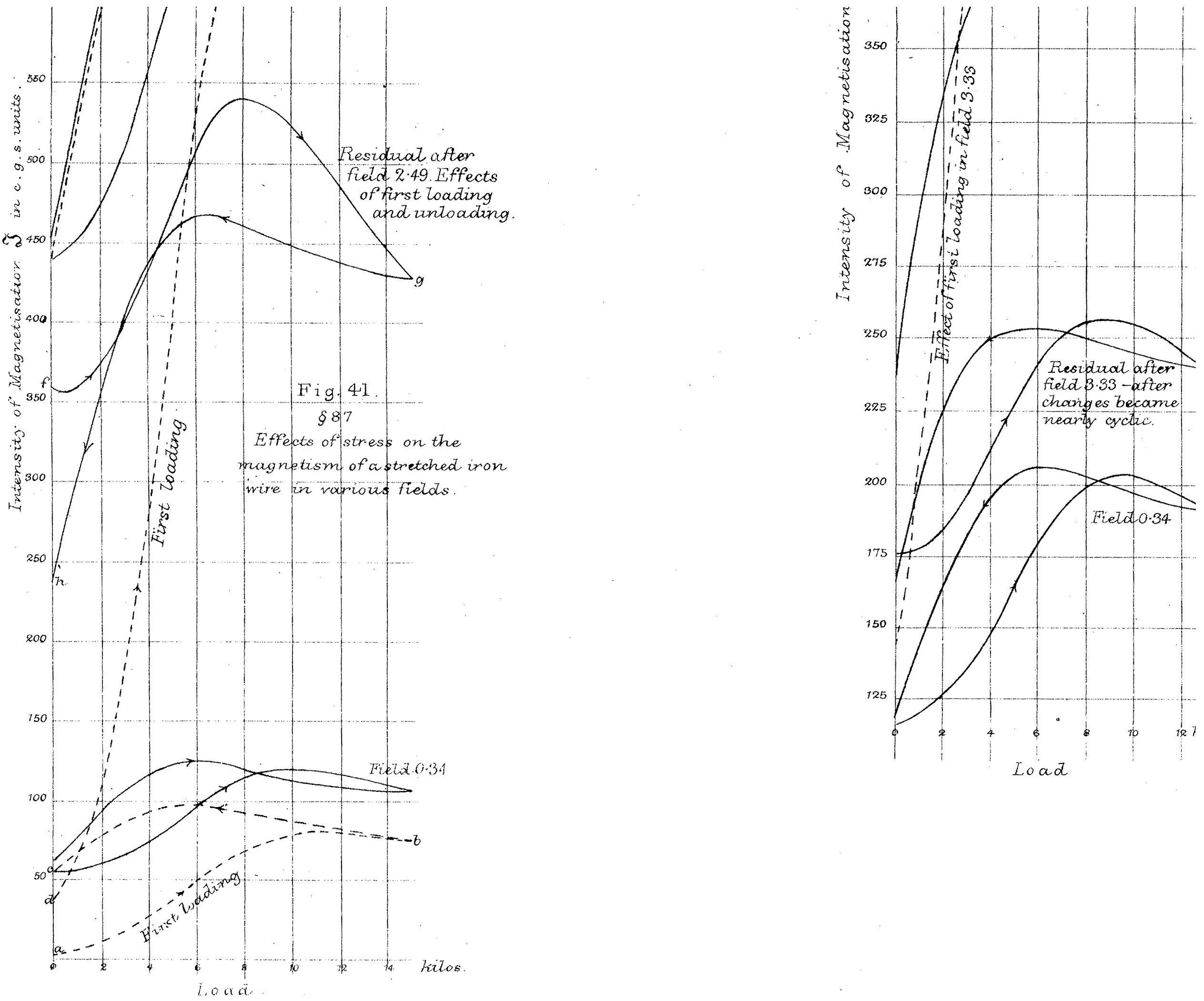
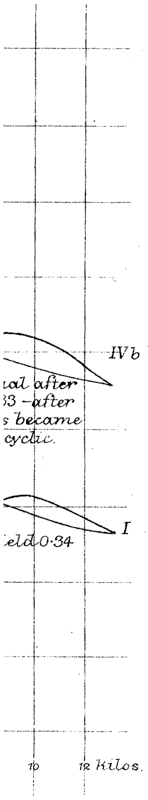
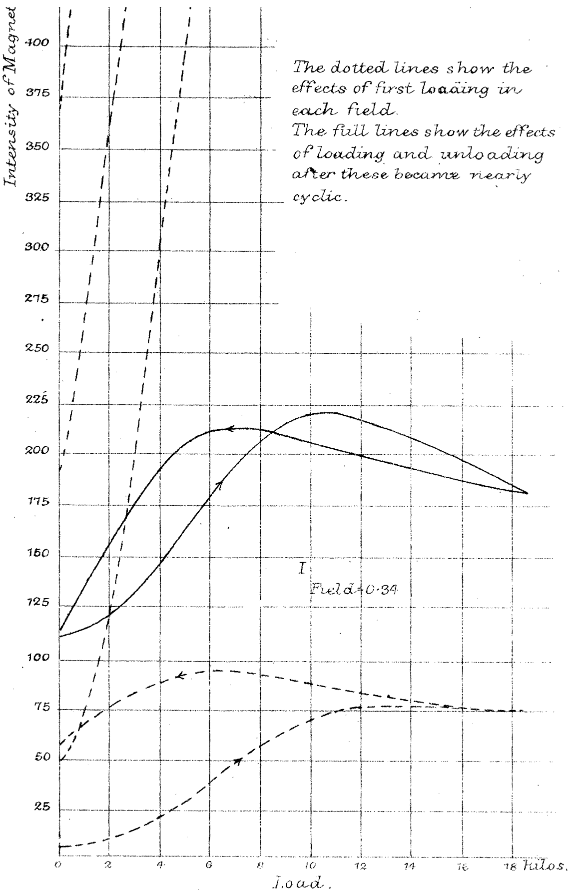


Fig. 41.
§ 87
Effects of stress on the magnetism of a stretched iron wire in various fields.


IVb
…al after
…33 – after
…s became
cyclic.
I
…ield 0·34
10
12 kilos.


Intensity of Magnet…
400
375
350
325
300
275
250
225
200
175
150
125
100
75
50
25
The dotted lines show the effects of first loading in each field.
The full lines show the effects of loading and unloading after these became nearly cyclic.
I
Field 0·34
0
2
4
6
8
10
12
14
16
18 kilos.
Load.

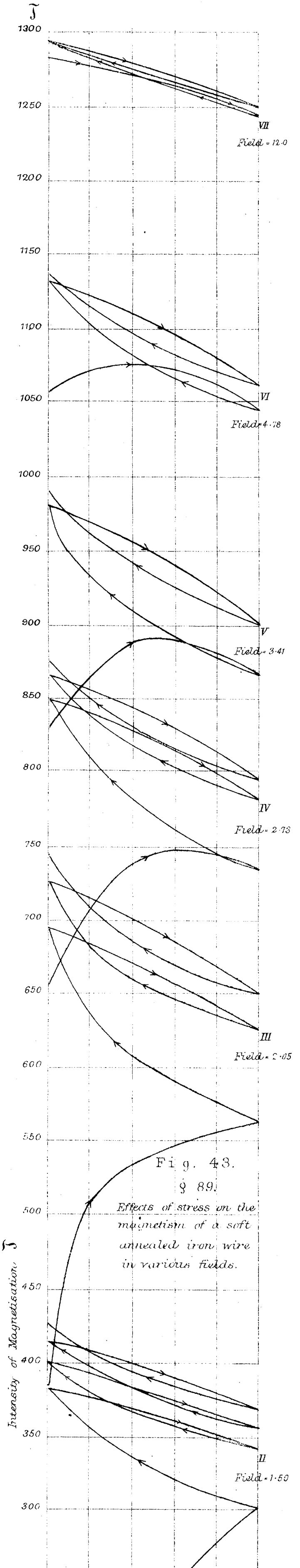
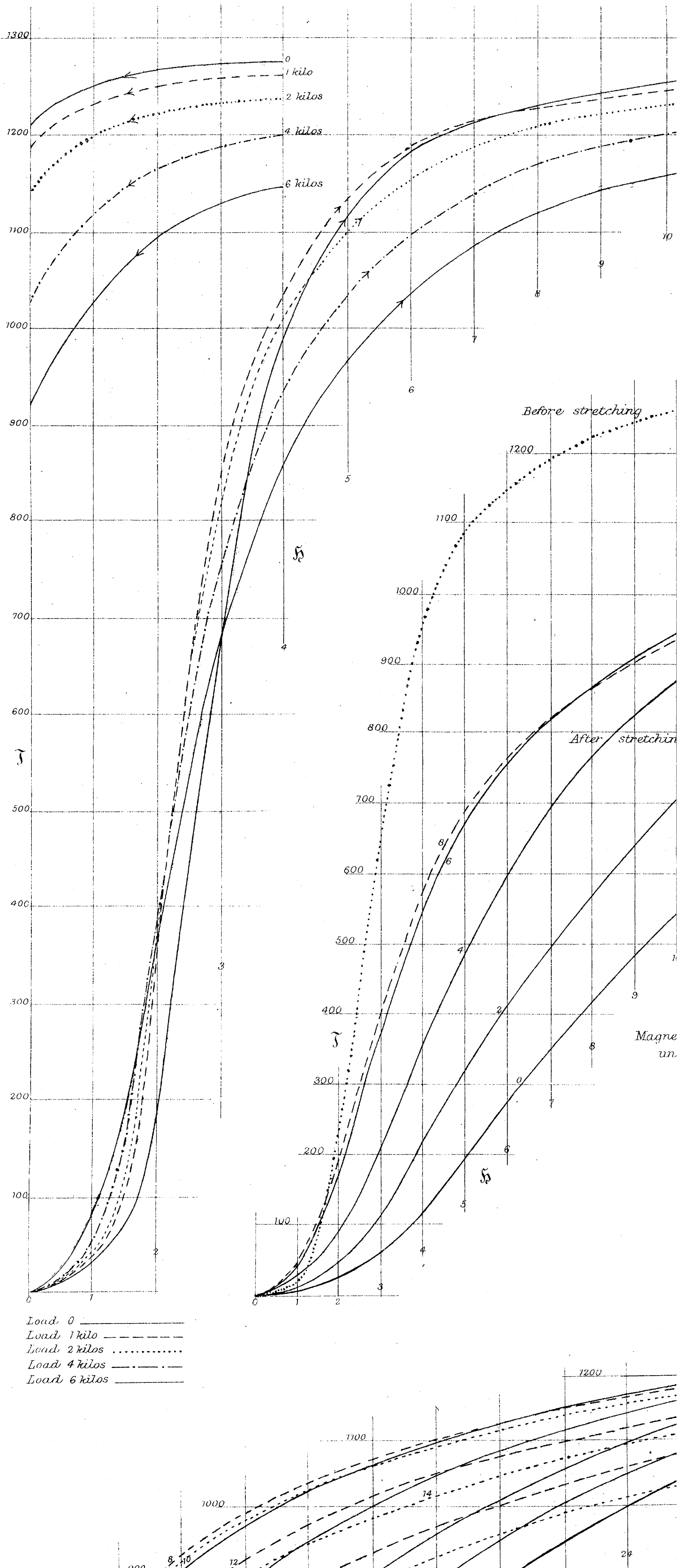


Fig. 43.

§ 89.

Effects of stress on the magnetism of a soft annealed iron wire in various fields.



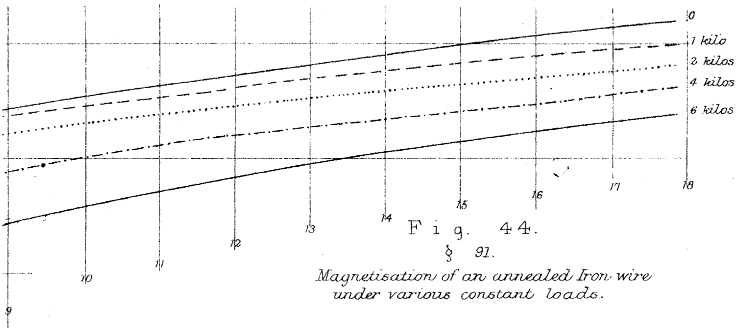


Fig. 44.

§ 91.

Magnetisation of an annealed Iron wire under various constant loads.

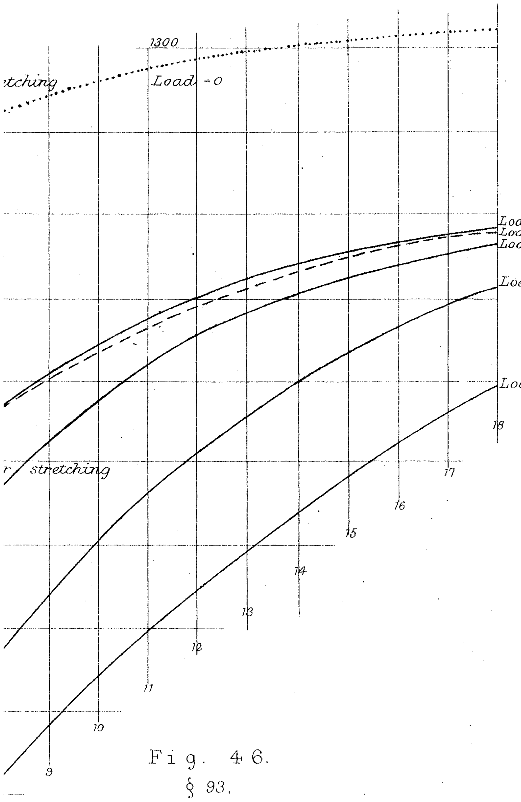


Fig. 46.

§ 93.

Magnetisation of a stretched iron wire under various constant loads.

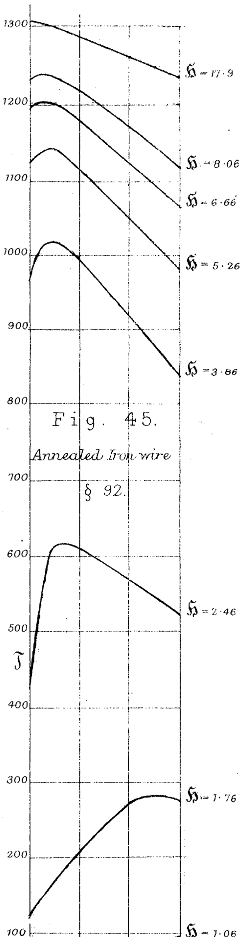
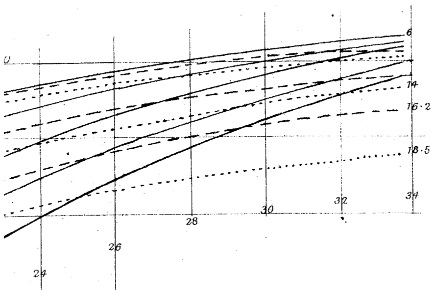


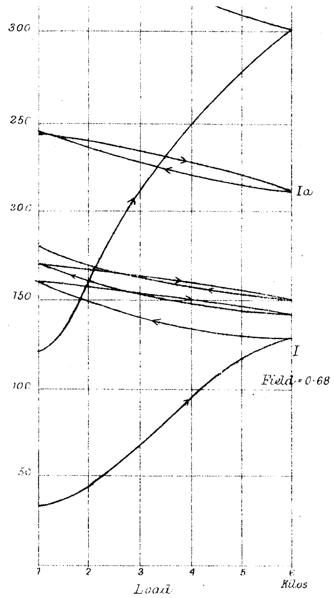
Fig. 45.

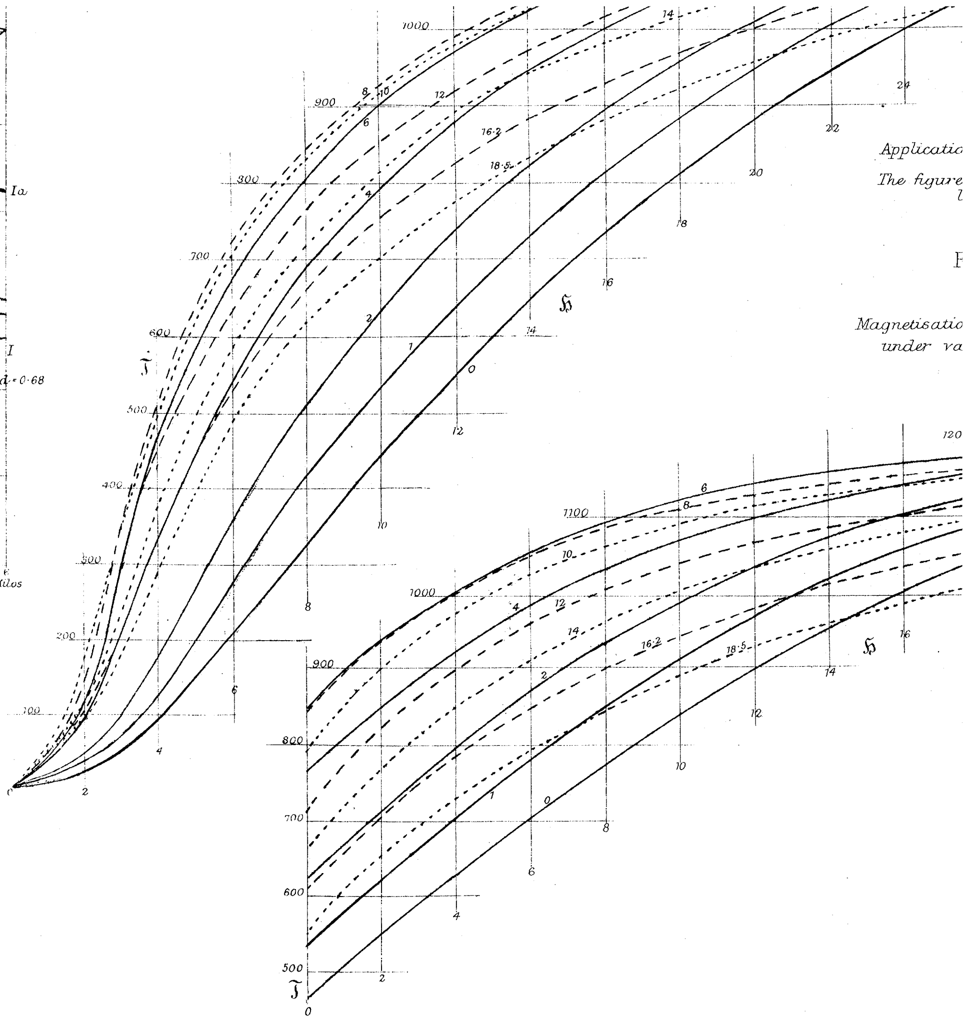
Annealed Iron wire

§ 92.



West, Newman & Cº lith.





Applicatio

The figure
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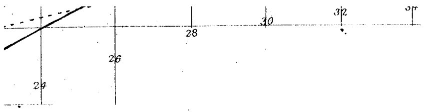
F

Magnetisatio
under va

Ewing.



Application of magnetising force.

The figures on the curves are the loads in kilos.

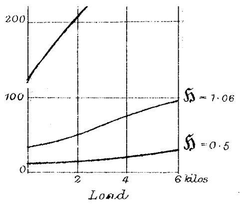
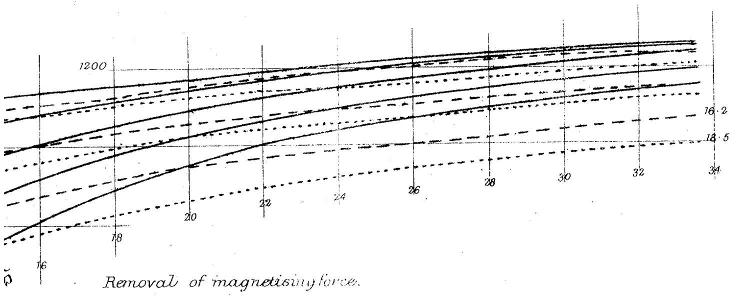


Fig. 47.

§ 94.

agnetisation of a stretched Iron wire under various constant loads.



Removal of magnetising force.

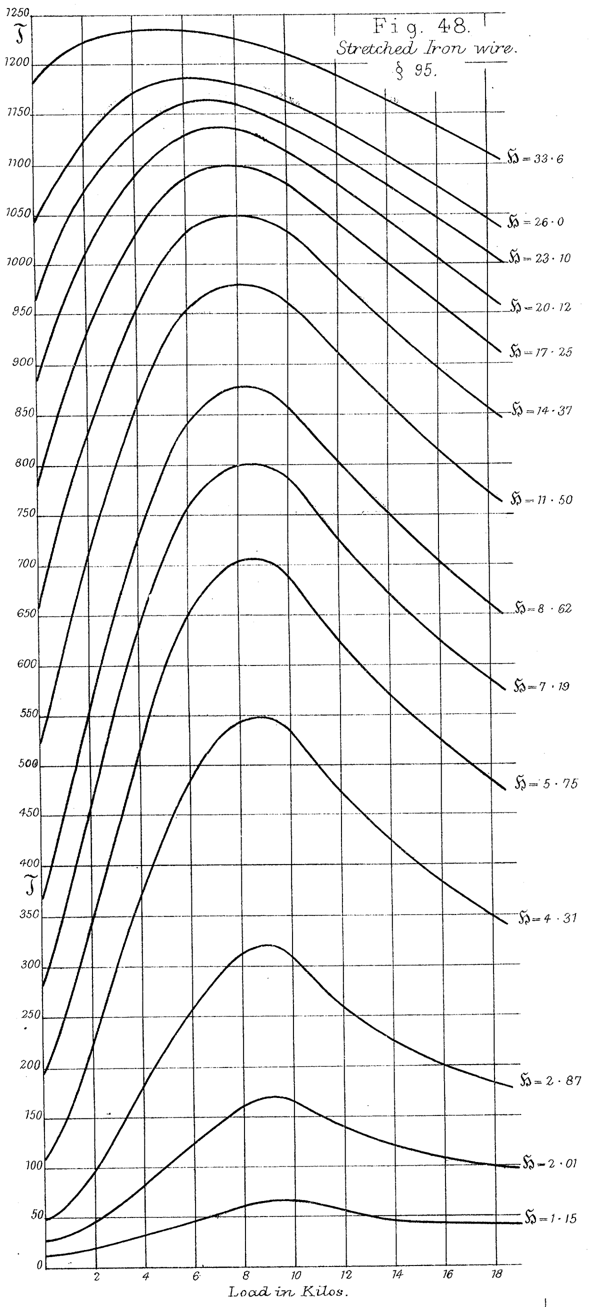


Fig. 48.
Stretched Iron wire.
§ 95.

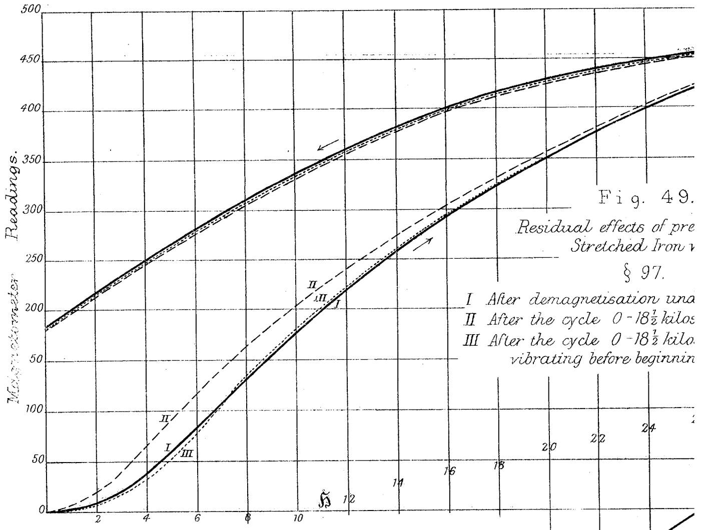


Fig. 49.
Residual effects of pre
Stretched Iron w
§ 97.
I After demagnetisation und
II After the cycle 0 - 18½ kilos
III After the cycle 0 - 18½ kilo
vibrating before beginnin

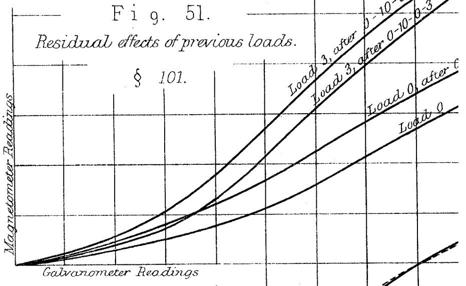
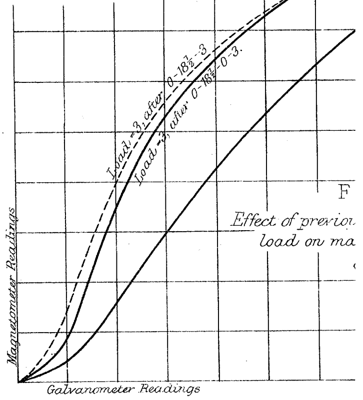
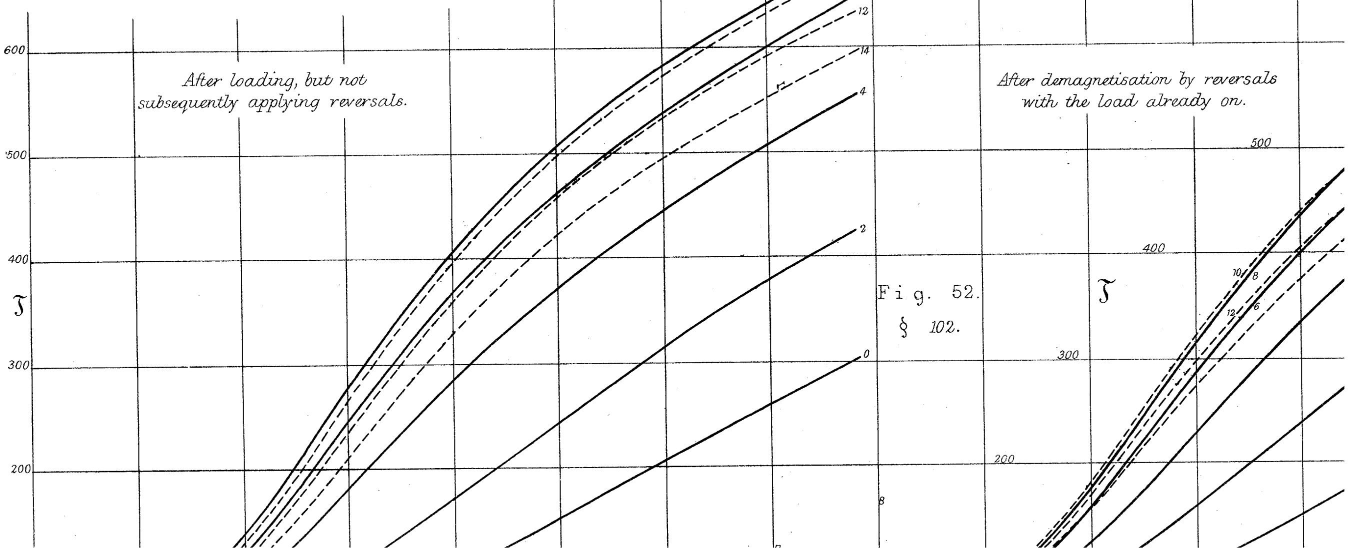


Fig. 51.
Residual effects of previous loads.
§ 101.



F
Effect of previou
load on ma



After loading, but not subsequently applying reversals.

After demagnetisation by reversals with the load already on.

Fig. 52.
§ 102.

ans. 1885. *Plate.* **65.**

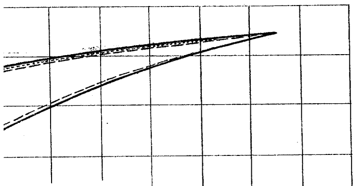


Fig. 49.

effects of previous Loads
tched Iron wire.

§ 97.

isation under no load ————
0-18½ kilos -0 – – – – – –
0-18½ kilos -0, and then
efore beginning to magnetise.} - - - - - -

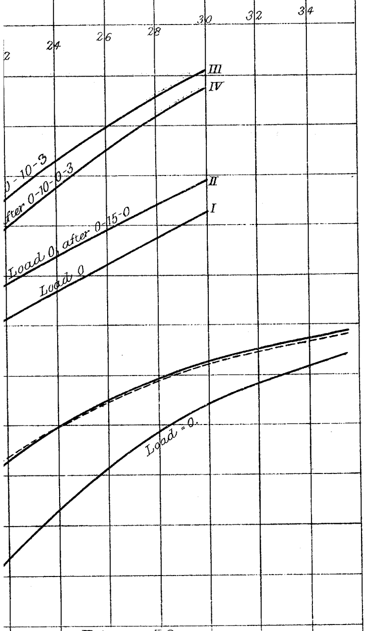
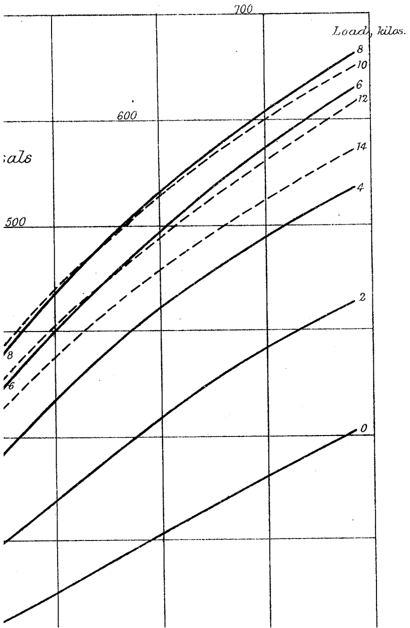
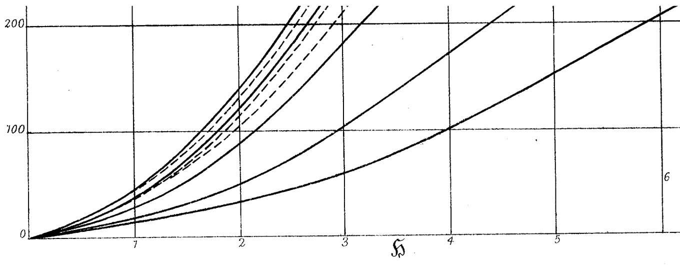


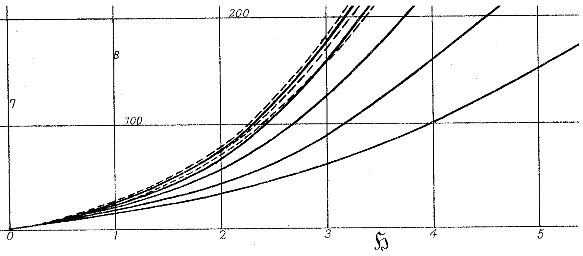
Fig. 50.

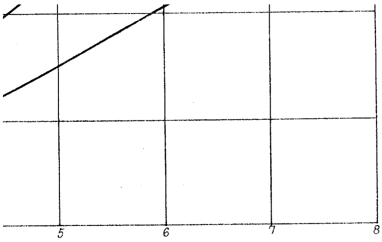
ct of previously applied and removed
load on magnetic susceptibility.

§ 100.




200
100
0
1
2
3
4
5
6
ℌ


7
8
200
100
0
1
2
3
4
5
ℌ



West Newman & C° lith.

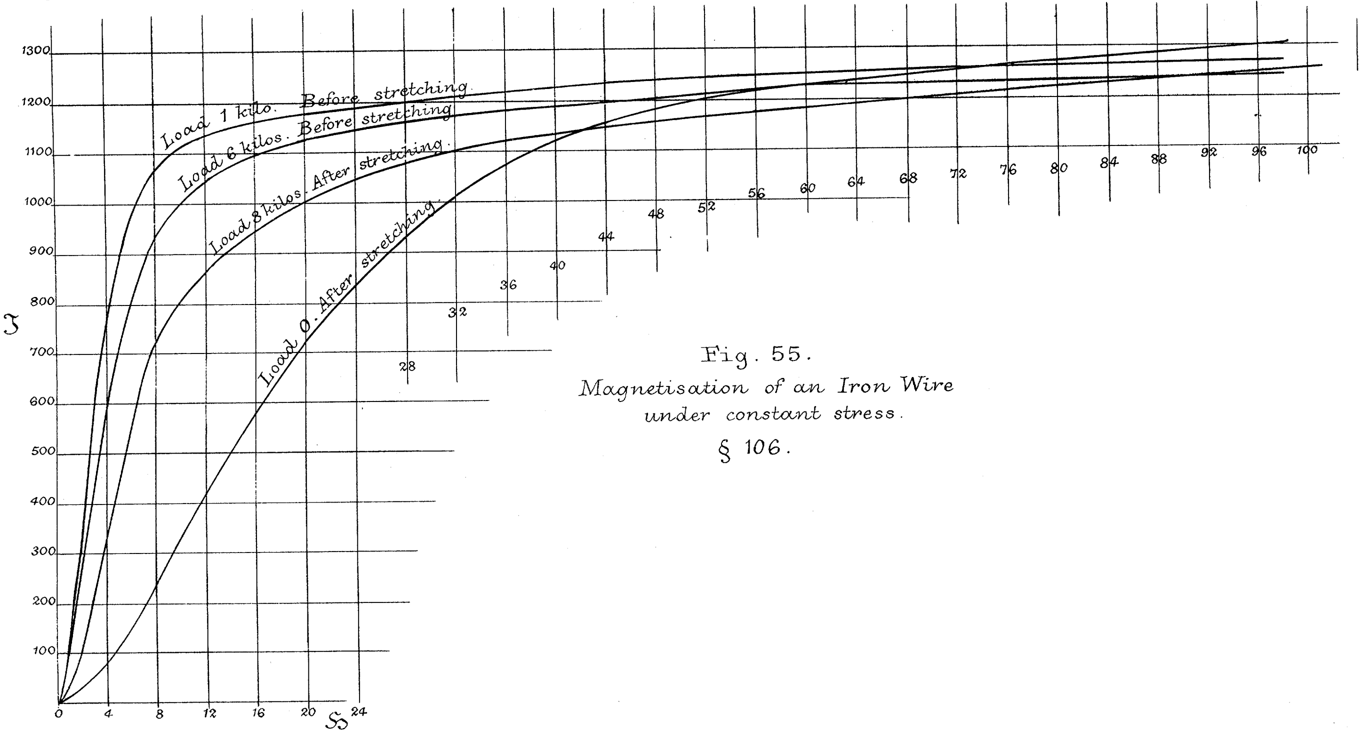


Fig. 55.

Magnetisation of an Iron Wire under constant stress.

§ 106.

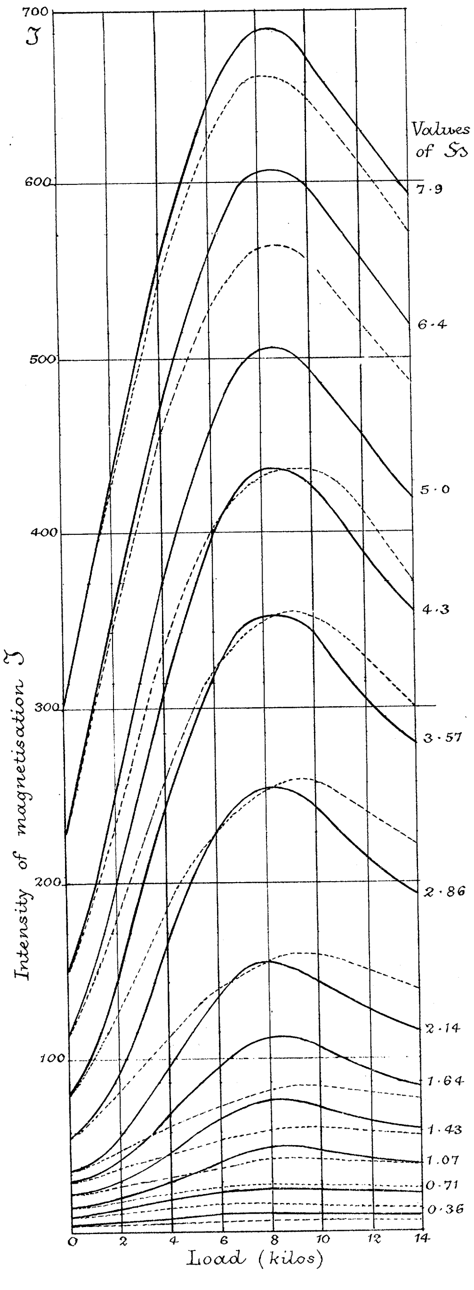


Fig. 53.

Magnetisation of a stretched Iron Wire under constant stress.

§ 103.

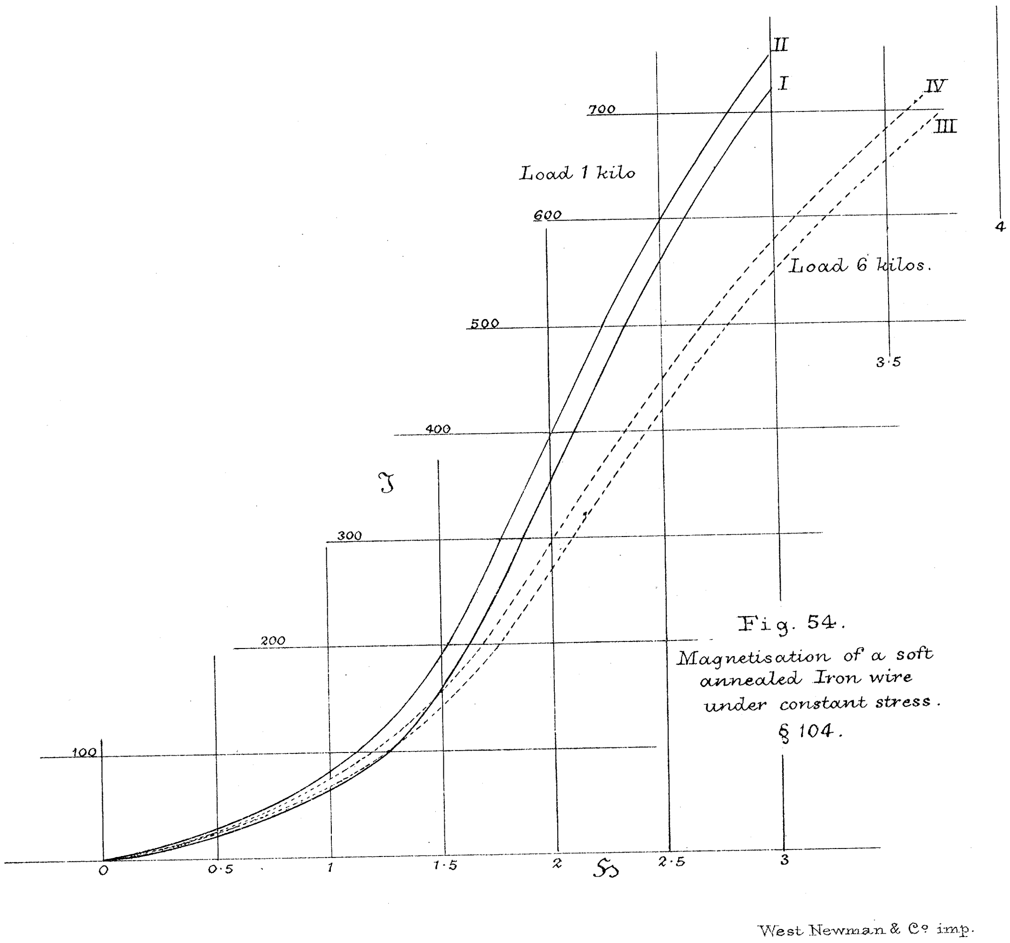


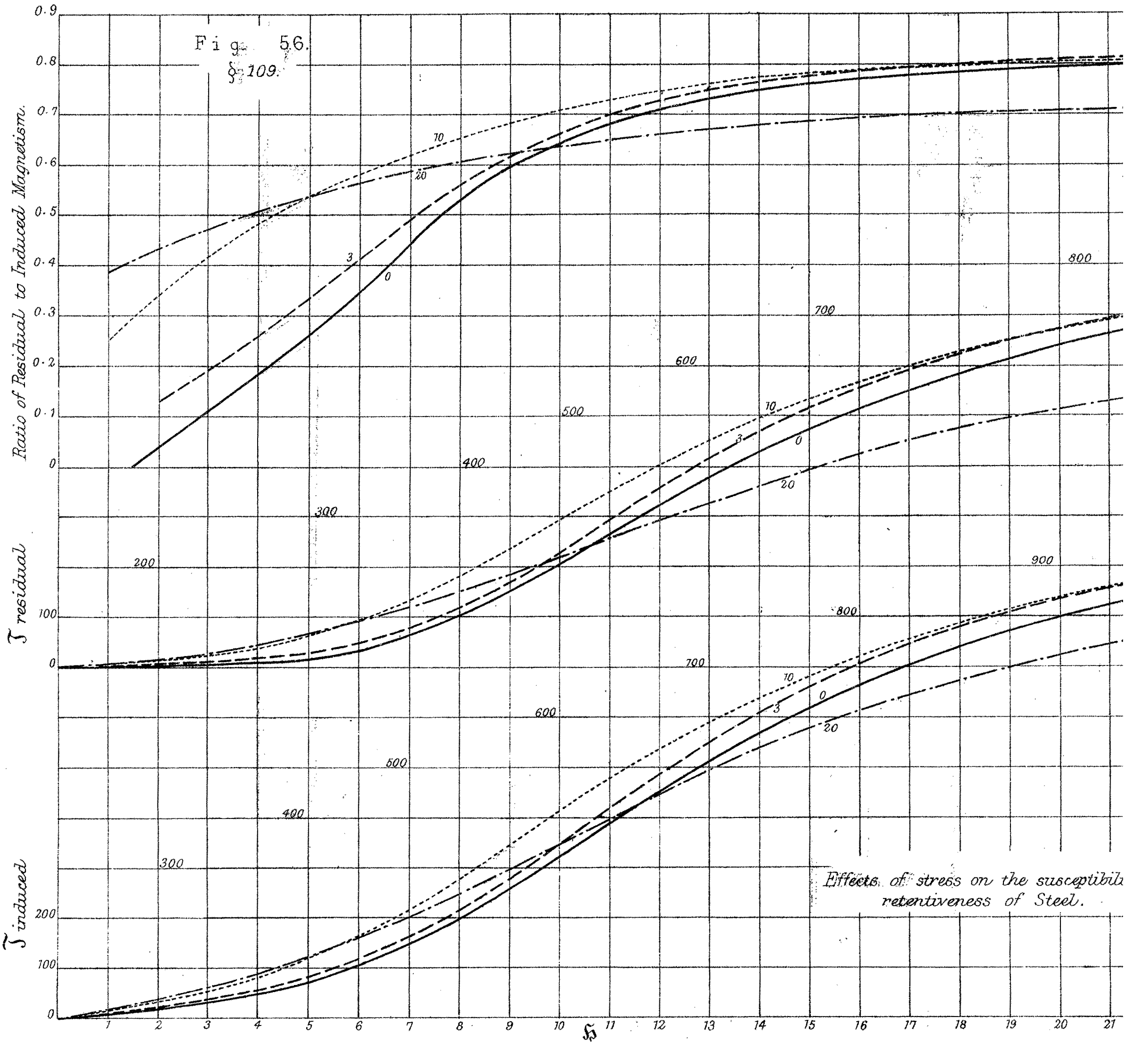
Fig. 54.

Magnetisation of a soft annealed Iron wire under constant stress.

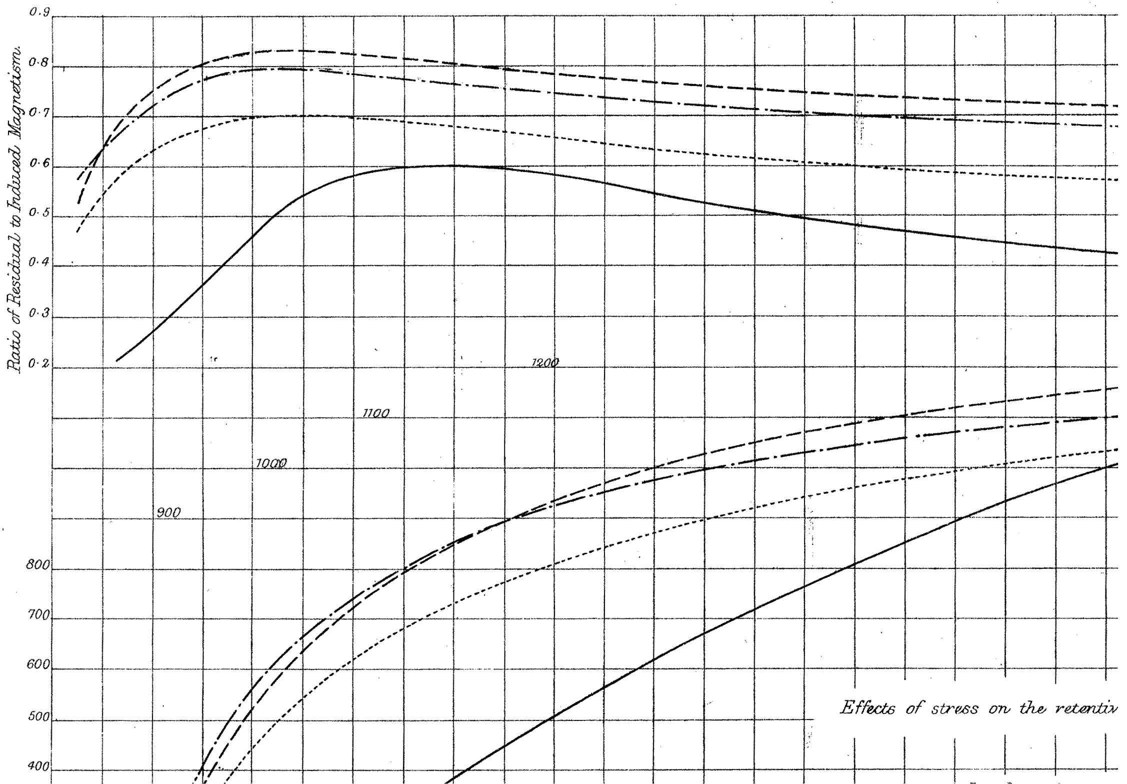
§ 104.

West Newman & Co. imp.

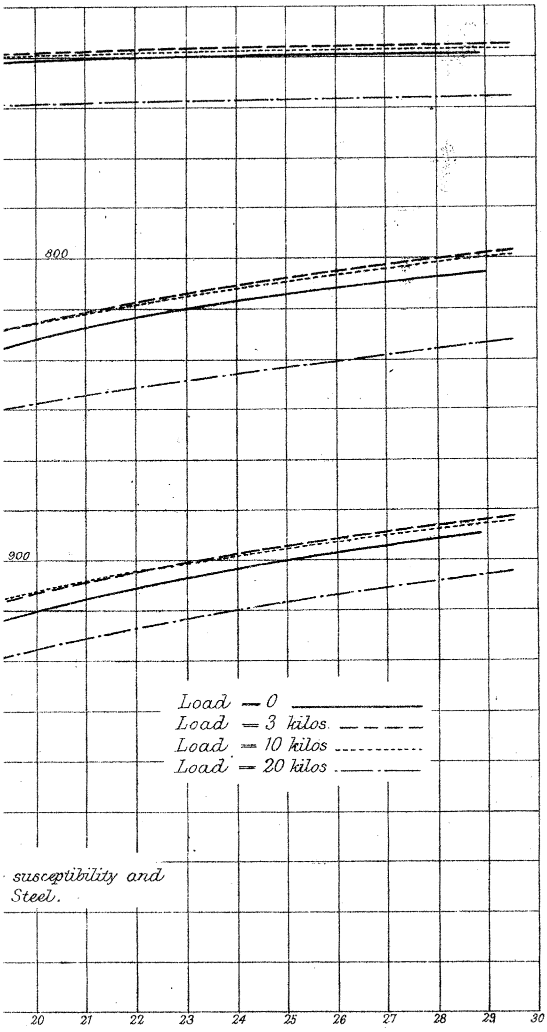
Fig. 56.
§ 109.



Effects of stress on the susceptibili
retentiveness of Steel.

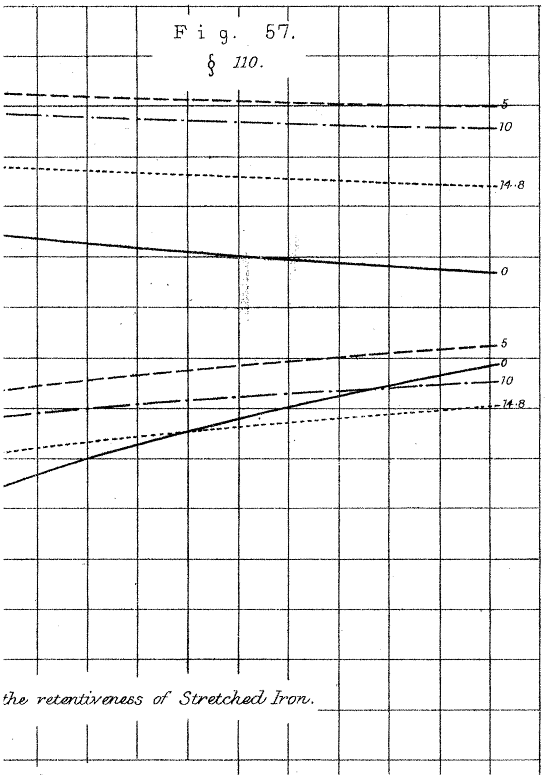


Effects of stress on the retentiv



susceptibility and Steel.

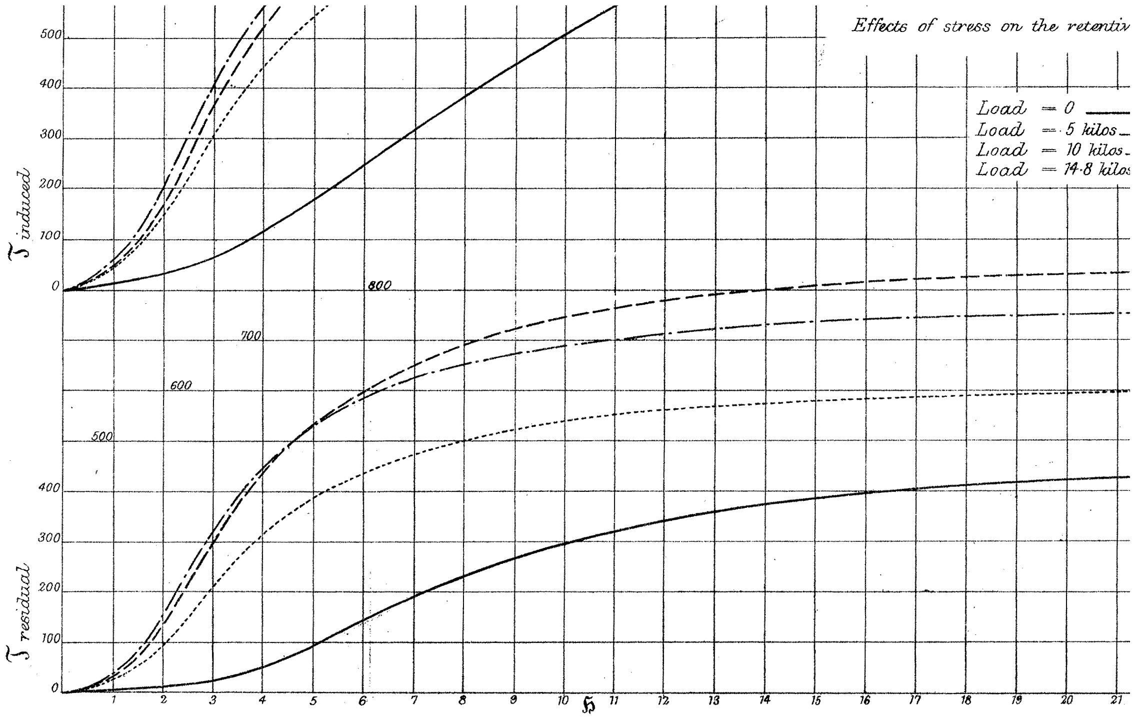
Fig. 57.
§ 110.



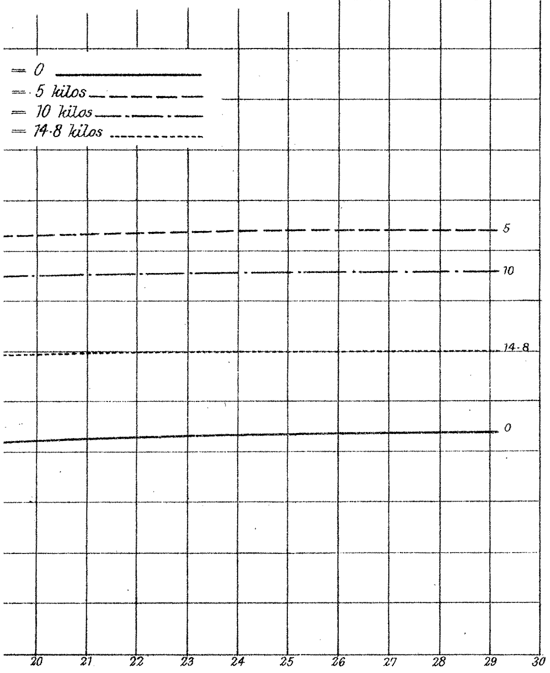
the retentiveness of Stretched Iron.

West Newman & Cº lith.

Ewing.


Effects of stress on the retentiv

the retentiveness of Stretched Iron.



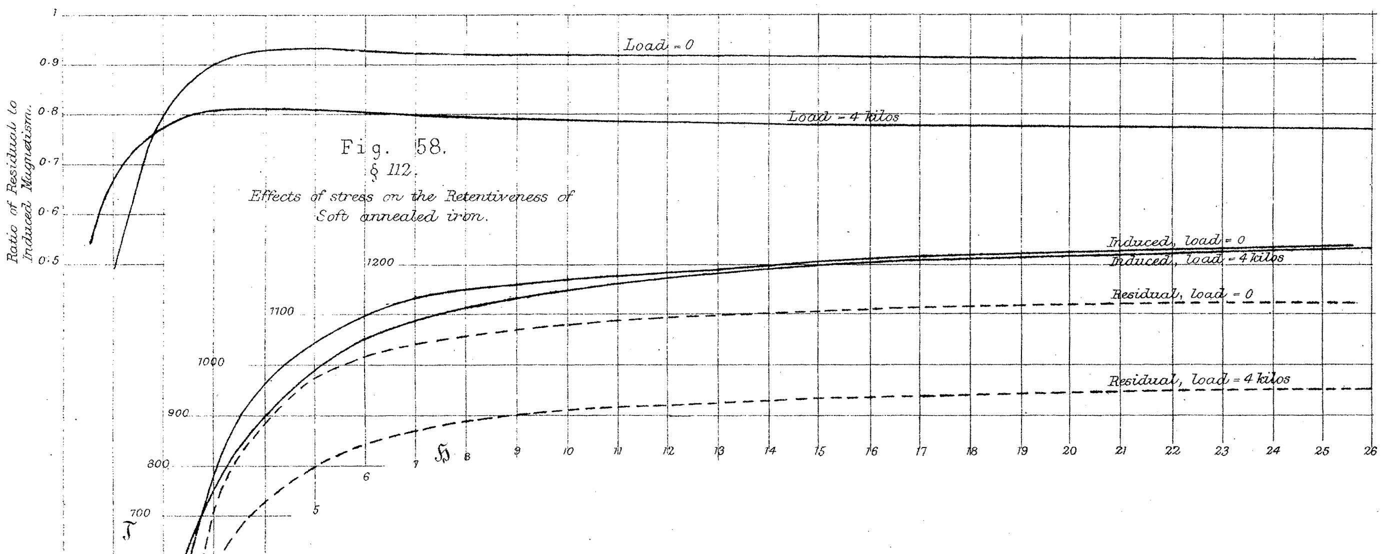


Fig. 58.
§ 112.
Effects of stress on the Retentiveness of Soft annealed iron.

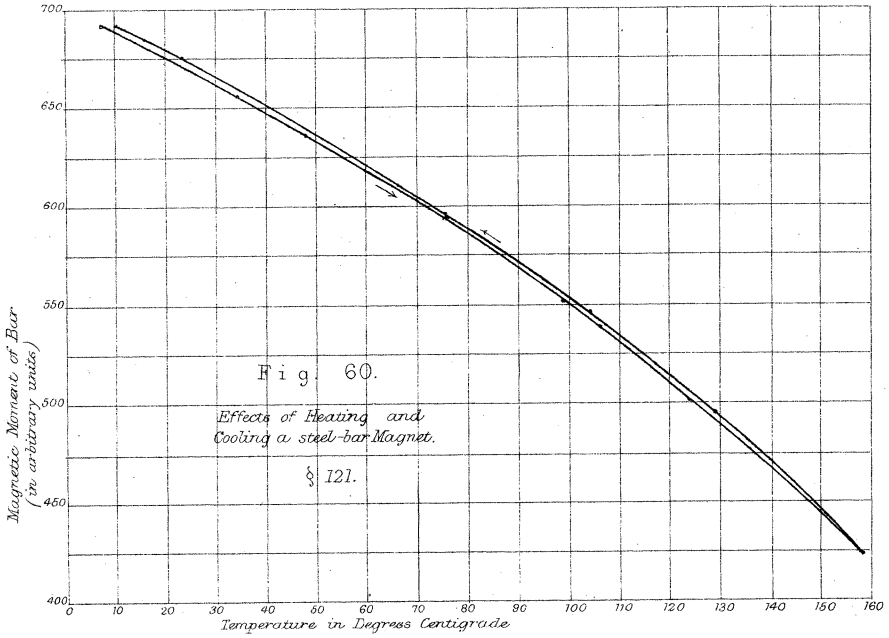


Fig. 60.
Effects of Heating and Cooling a steel-bar Magnet.
§ 121.

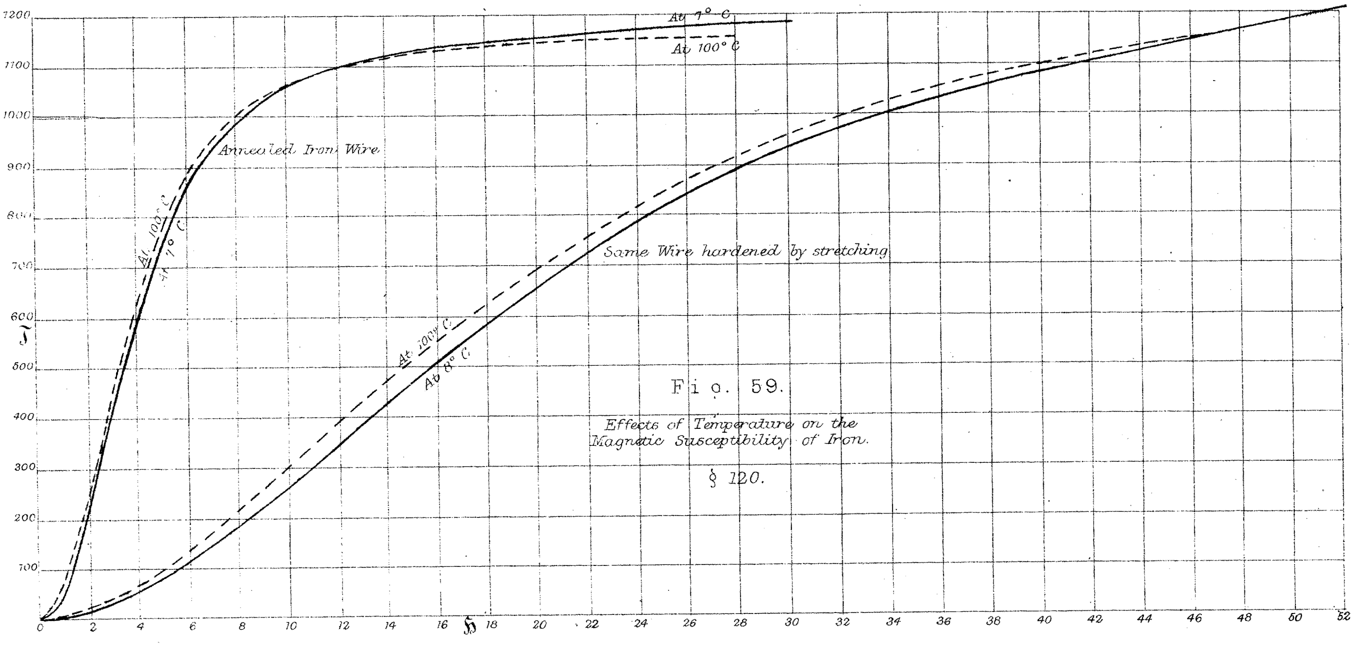
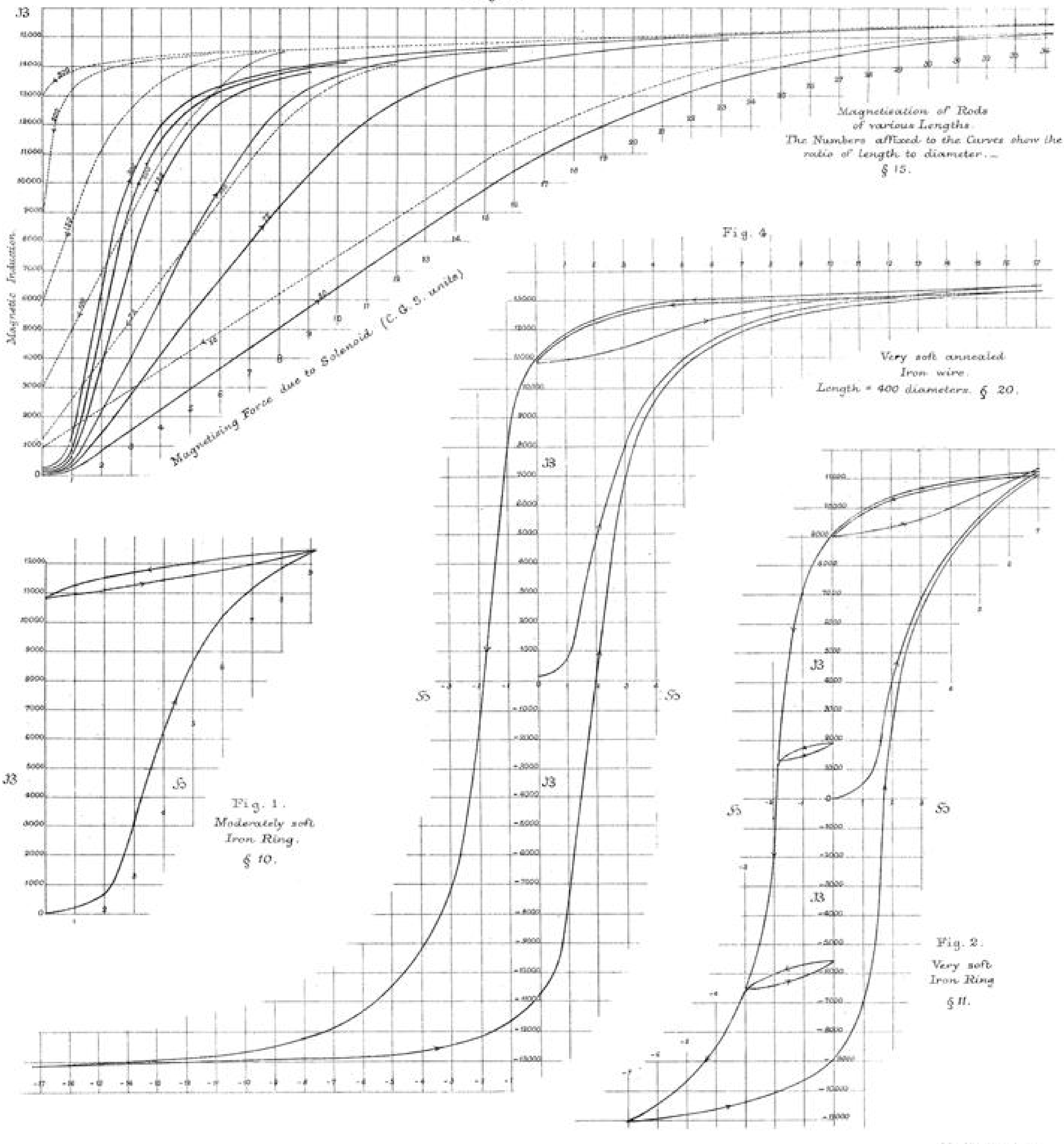


Fig. 59.
Effects of Temperature on the Magnetic Susceptibility of Iron.
§ 120.

West, Newman & Co. lith.

Fig. 3.



Magnetisation of Rods of various Lengths. The Numbers affixed to the Curves show the ratio of length to diameter.— § 15.

Fig. 4.

Very soft annealed Iron wire. Length = 400 diameters. § 20.

Fig. 1.

Moderately soft Iron Ring. § 10.

Fig. 2.

Very soft Iron Ring § 11.

West, Newman & Co lith.

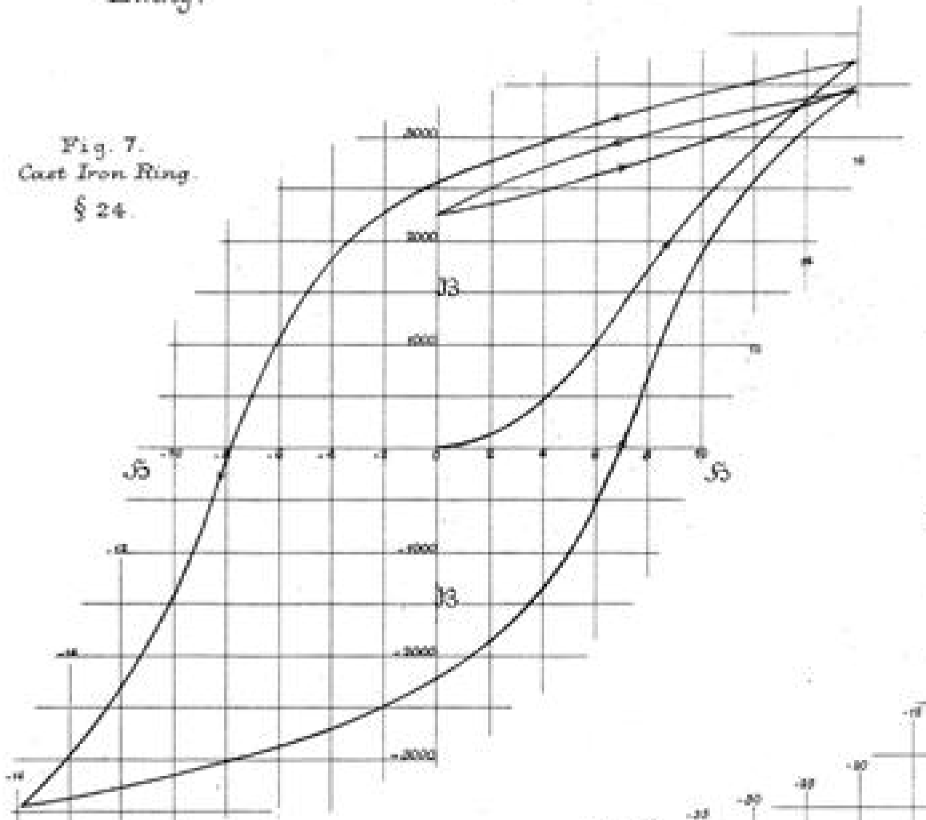


Fig. 7.
Cast Iron Ring.
§ 24.

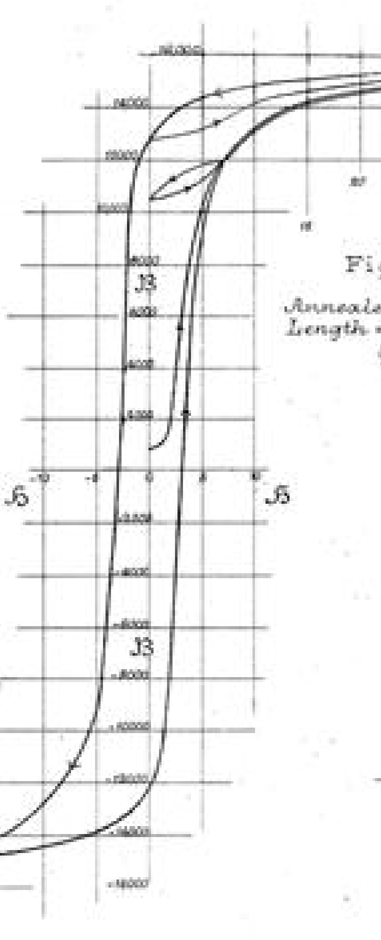


Fig. 6.
Annealed Iron wire
Length = 400 diam.s
§ 22.

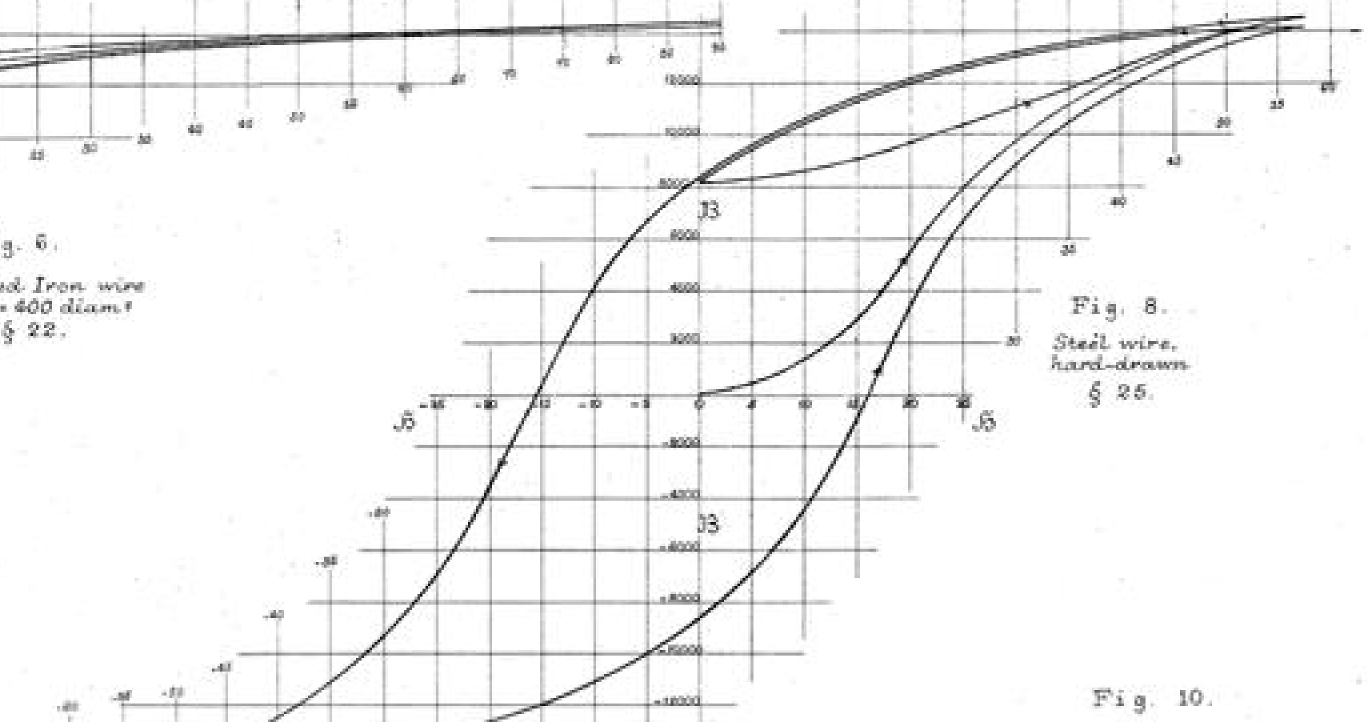


Fig. 8.
Steel wire,
hard-drawn
§ 25.

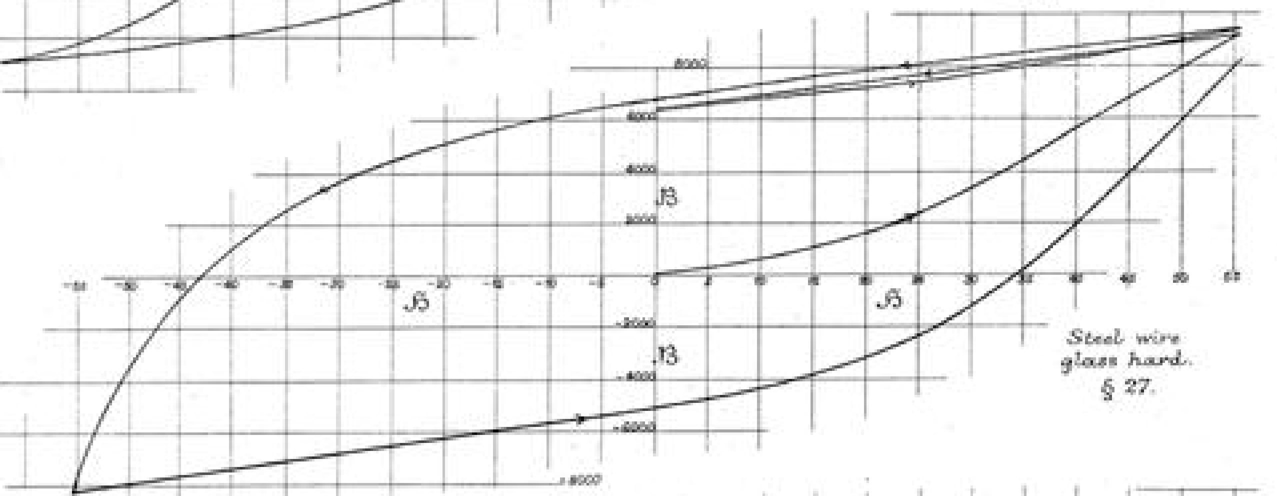


Fig. 10.
Steel wire
glass hard.
§ 27.

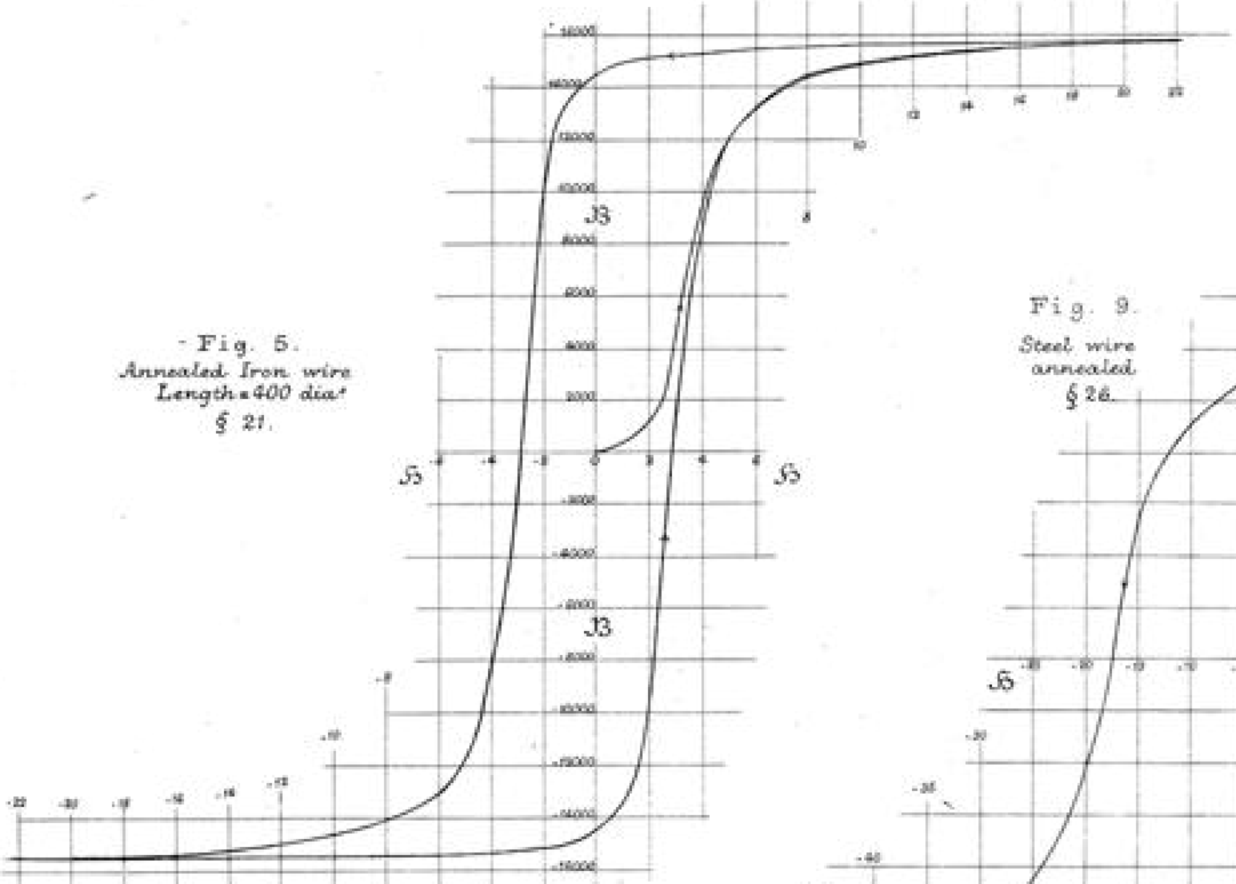


Fig. 5.
Annealed Iron wire
Length = 400 dia.s
§ 21.

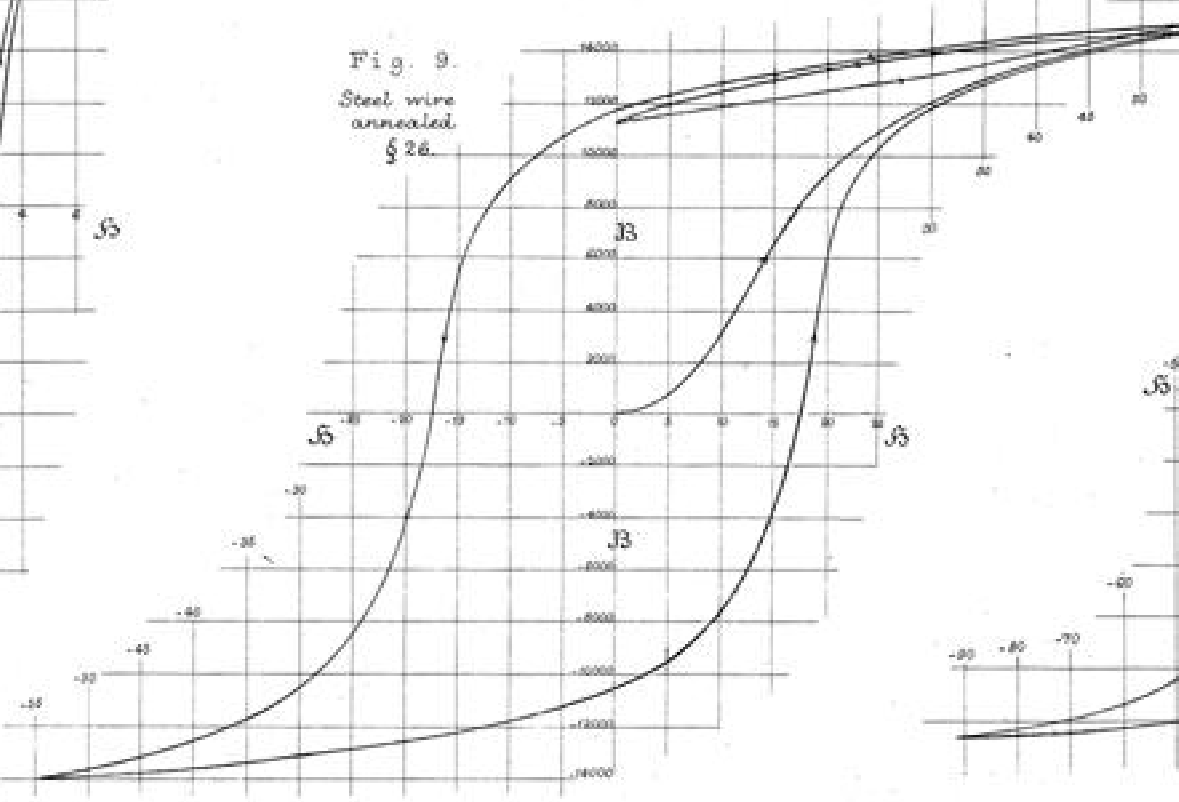


Fig. 9.
Steel wire
annealed
§ 26.

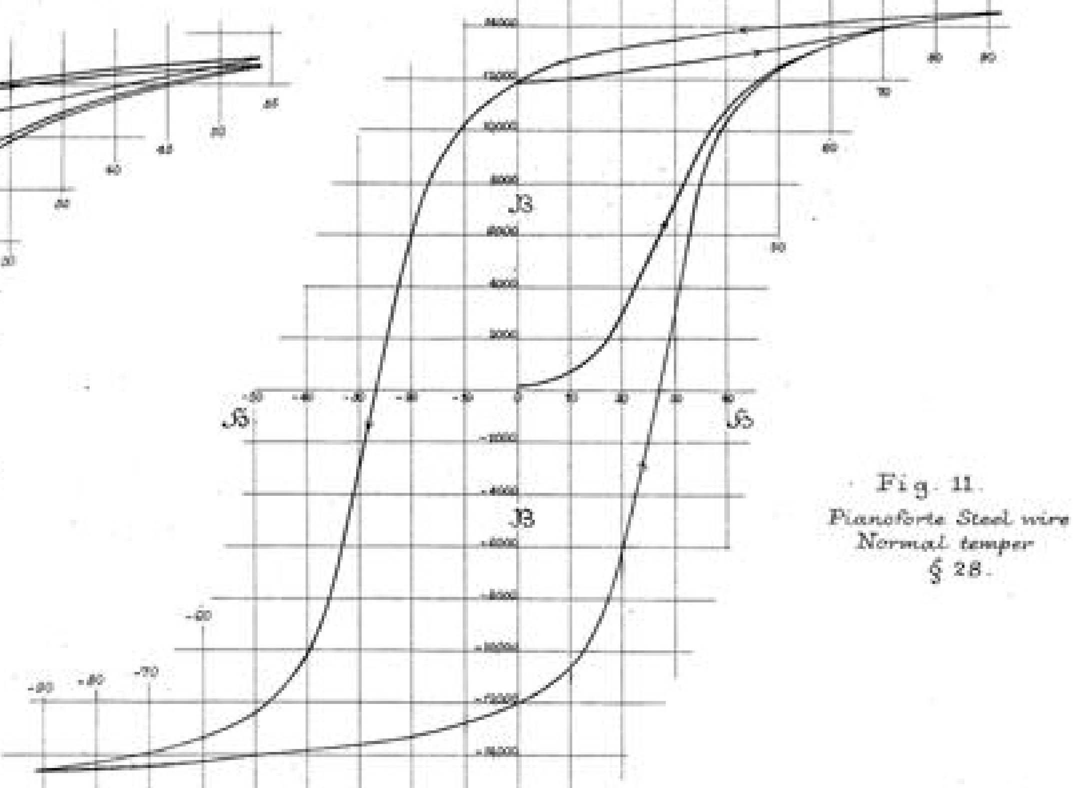


Fig. 11.
Pianoforte Steel wire
Normal temper
§ 28.

West, Newman & Co. lith.

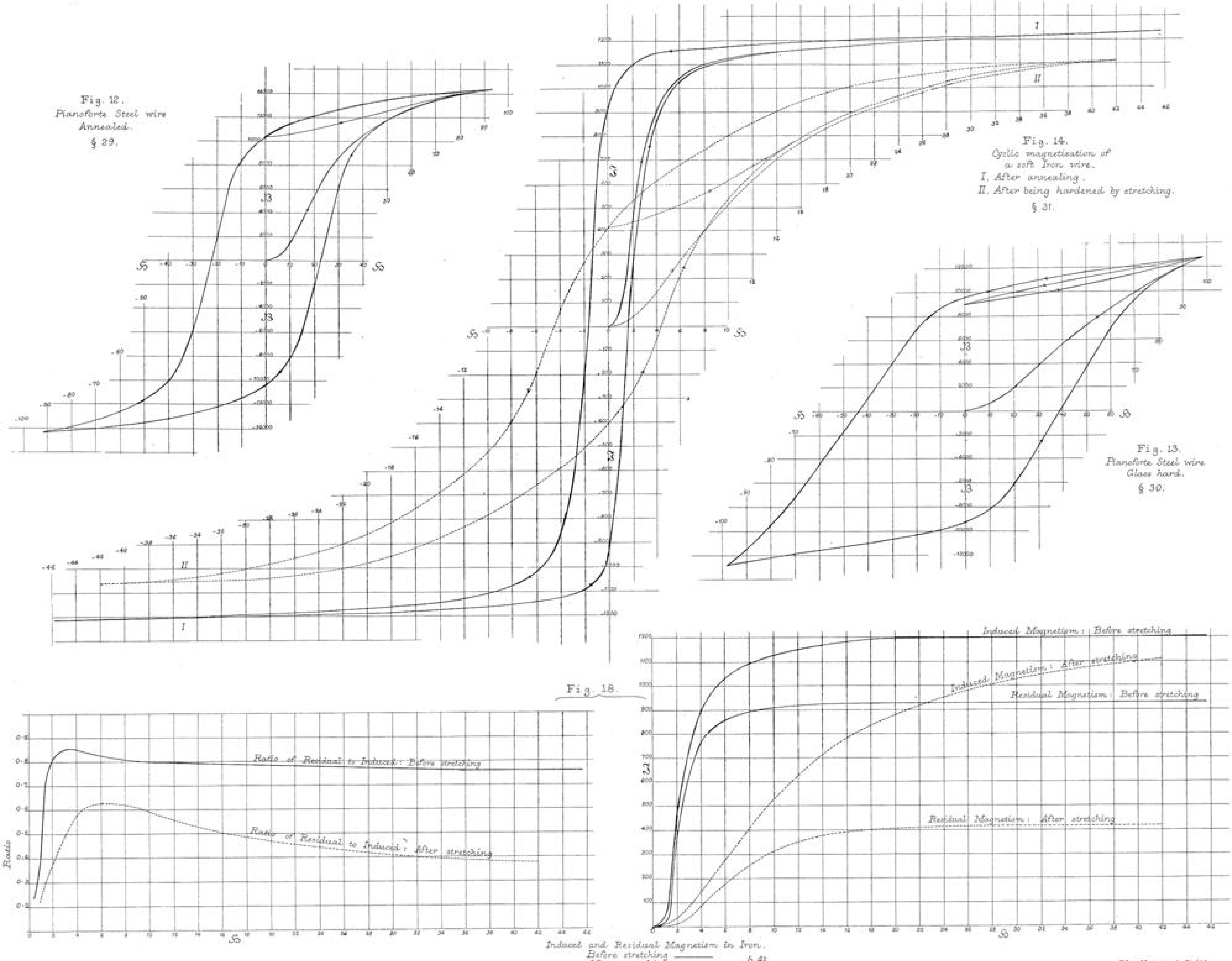


Fig. 12.
Pianoforte Steel wire
Annealed.
§ 29.

Fig. 14.
Cyclic magnetisation of
a soft Iron wire.
I. After annealing.
II. After being hardened by stretching.
§ 31.

Fig. 13.
Pianoforte Steel wire
Glass hard.
§ 30.

Fig. 18.

Induced and Residual Magnetism in Iron.
Before stretching ————
After stretching - - - - - -
§ 41.

West Newman & Co lith.

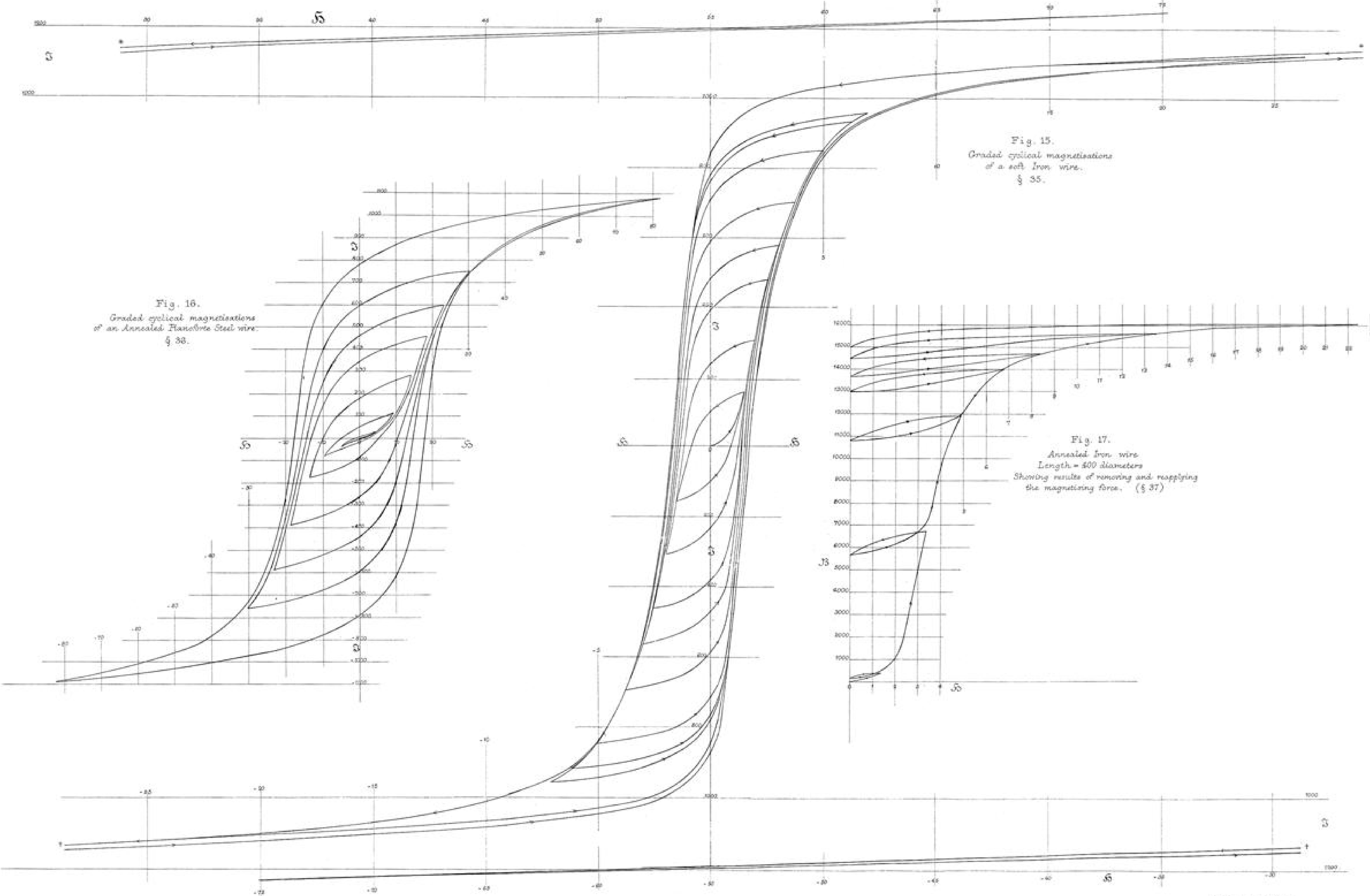


Fig. 15.
Graded cyclical magnetisations
of a soft Iron wire.
§ 35.

Fig. 16.
Graded cyclical magnetisations
of an Annealed Pianoforte Steel wire.
§ 38.

Fig. 17.
Annealed Iron wire
Length = 400 diameters
Showing results of removing and reapplying
the magnetising force. (§ 37)

West, Newman & Co lith.

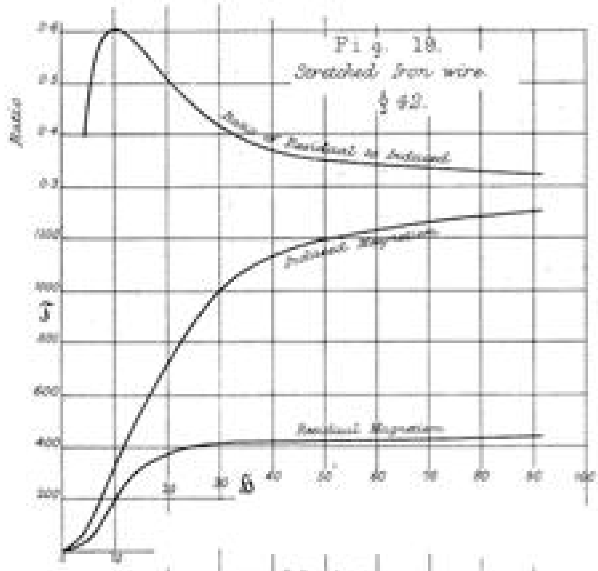


Fig. 19.
Stretched Iron wire.
§ 42.

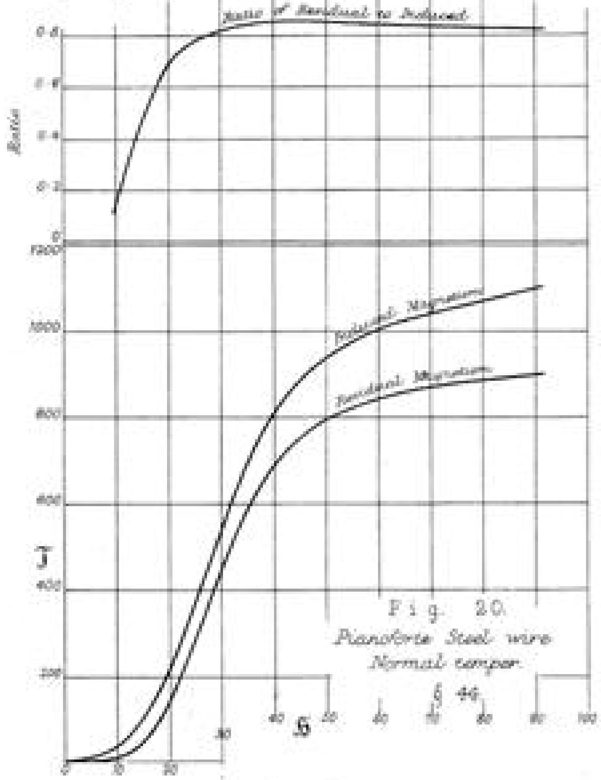


Fig. 20.
Pianoforte Steel wire
Normal temper.
§ 44.

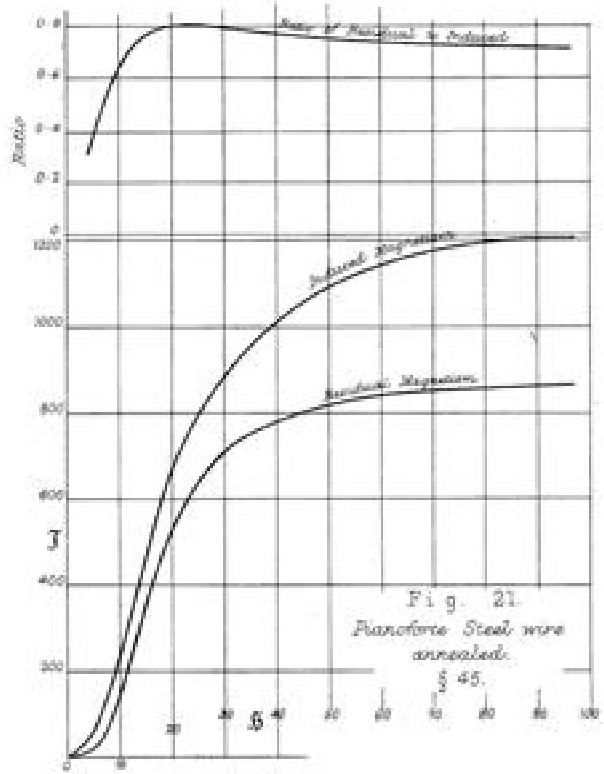


Fig. 21.
Pianoforte Steel wire
annealed.
§ 45.

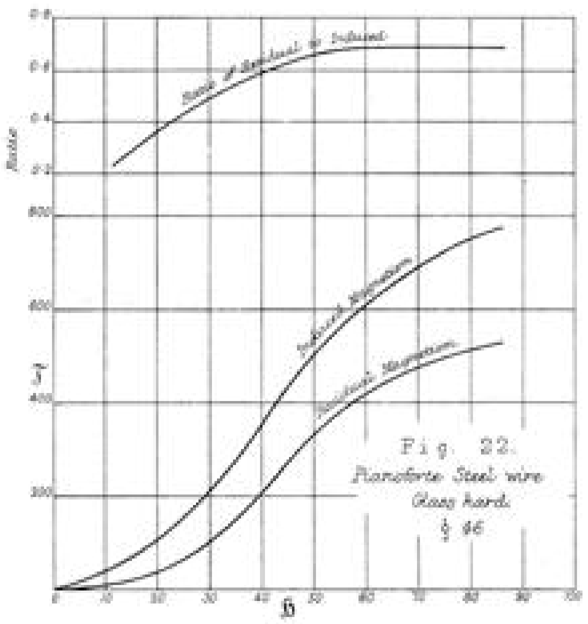


Fig. 22.
Pianoforte Steel wire
Glass hard.
§ 46.

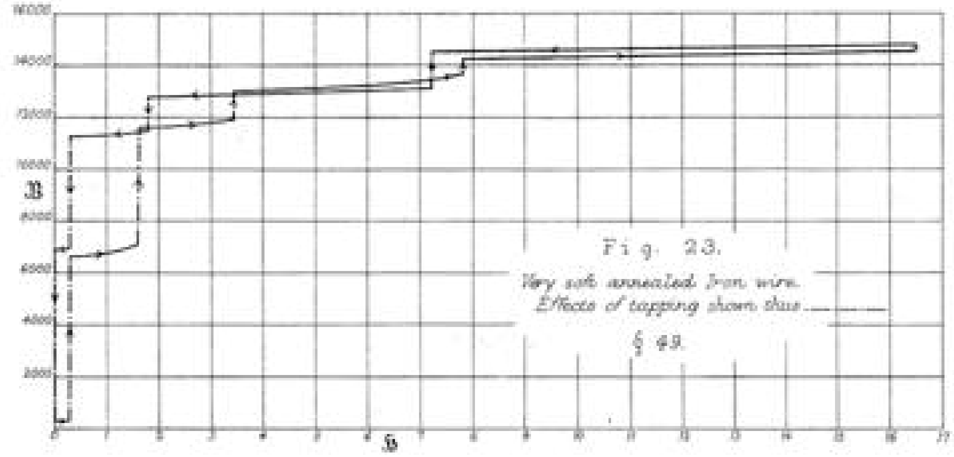


Fig. 23.
Very soft annealed Iron wire.
Effects of tapping shown thus -.-.-.-

§ 49.

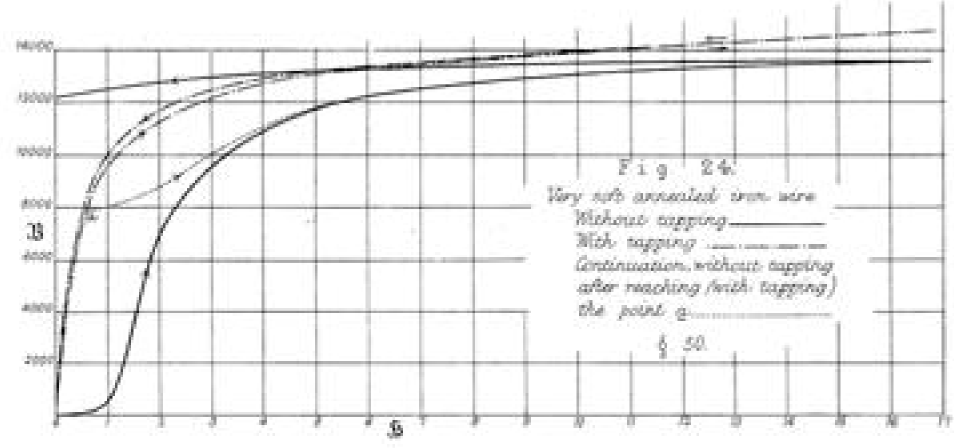


Fig. 24.
Very soft annealed iron wire
Without tapping ————
With tapping -.-.-.-.-
Continuation, without tapping
after reaching (with tapping)
the point a ....

§ 50.

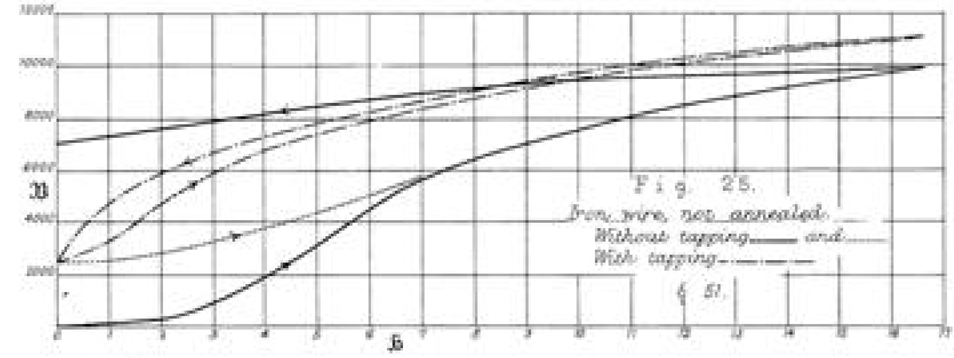


Fig. 25.
Iron wire, not annealed.
Without tapping ———— and ....
With tapping -.-.-.-.-

§ 51.

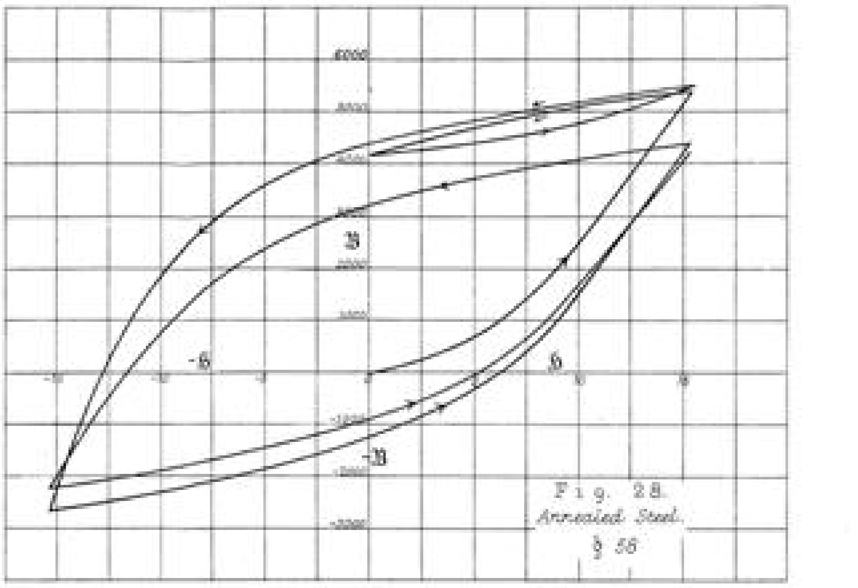


Fig. 28.
Annealed Steel.
§ 58.

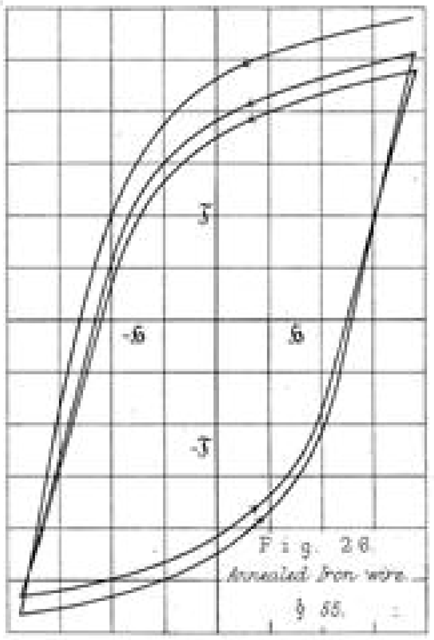


Fig. 26.
Annealed Iron wire.
§ 55.

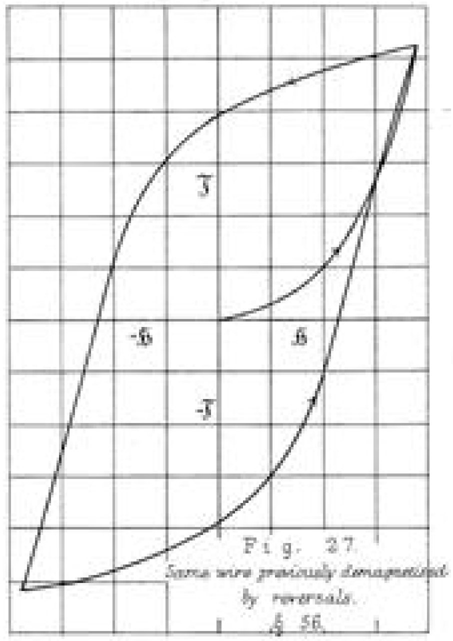


Fig. 27.
Same wire previously demagnetised
by reversals.
§ 56.

West Newman & Co lith.

Ewing.

Phil. Trans. 1885. Plate 62.

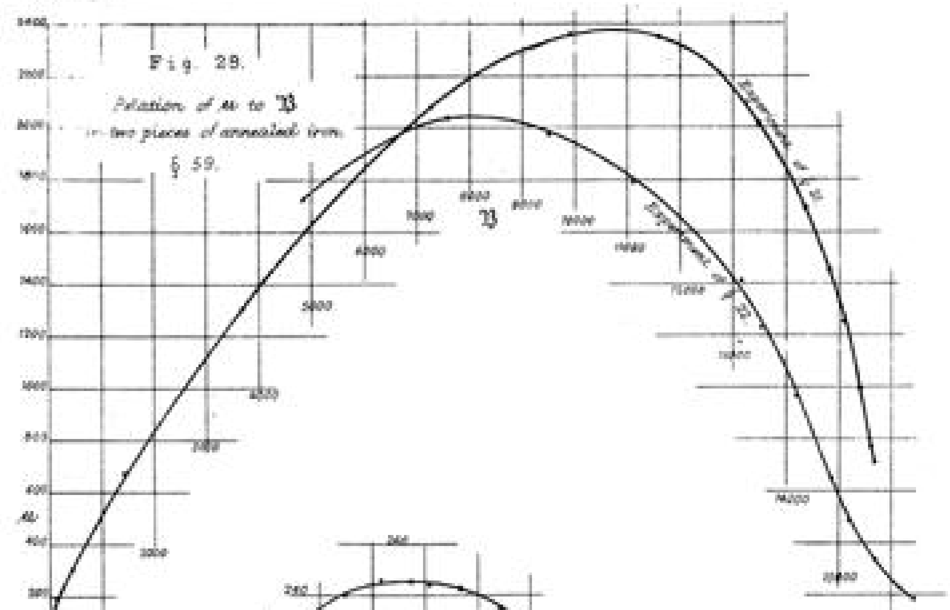


Fig. 29.
Relation of μ to $\mathfrak{B}$ in two pieces of annealed iron.
§ 59.

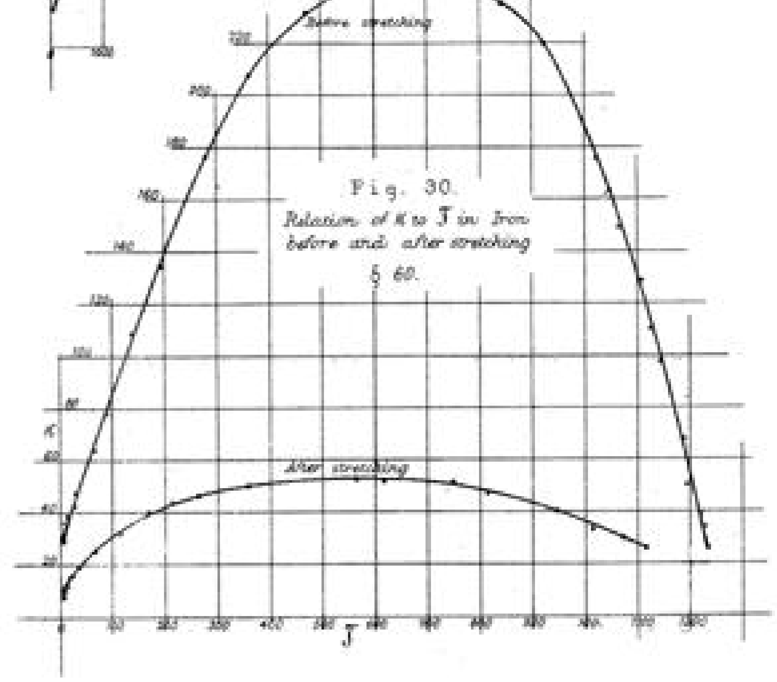


Fig. 30.
Relation of κ to $\mathfrak{I}$ in Iron before and after stretching
§ 60.

Effects of Stress on a Stretched Iron Wire in a Constant Magnetic Field (0·34 c.g.s. units).

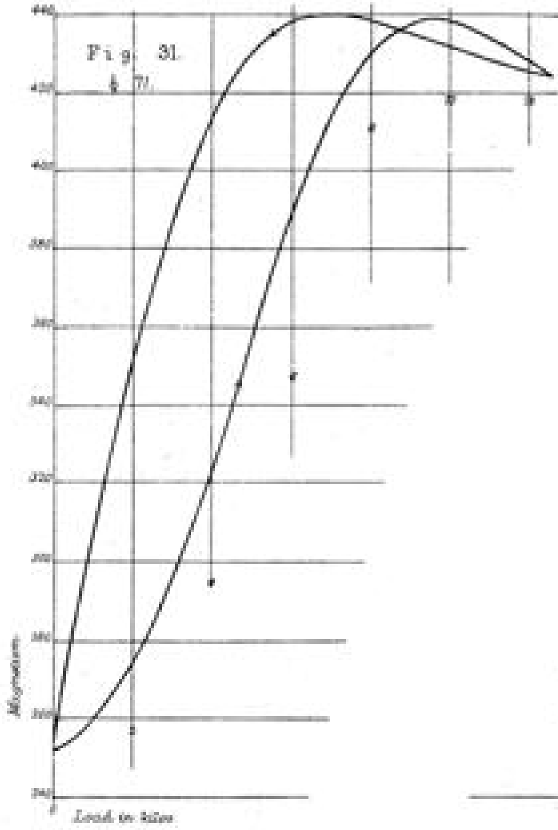


Fig. 31.
§ 71.

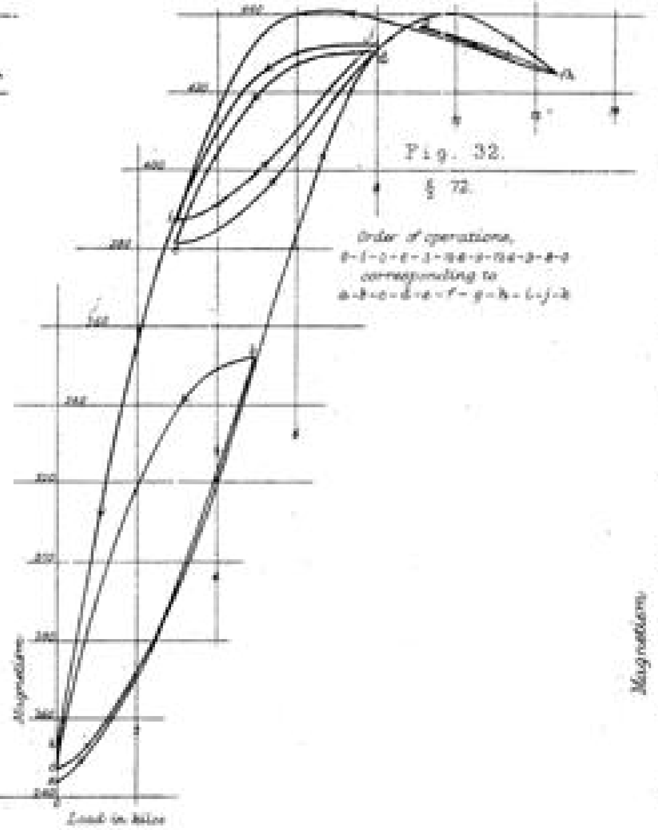


Fig. 32.
§ 72.
Order of operations,
0-1-2-4-3-12·6-3-12·6-3-8-0
corresponding to
a-b-c-d-e-f-g-h-i-j-k

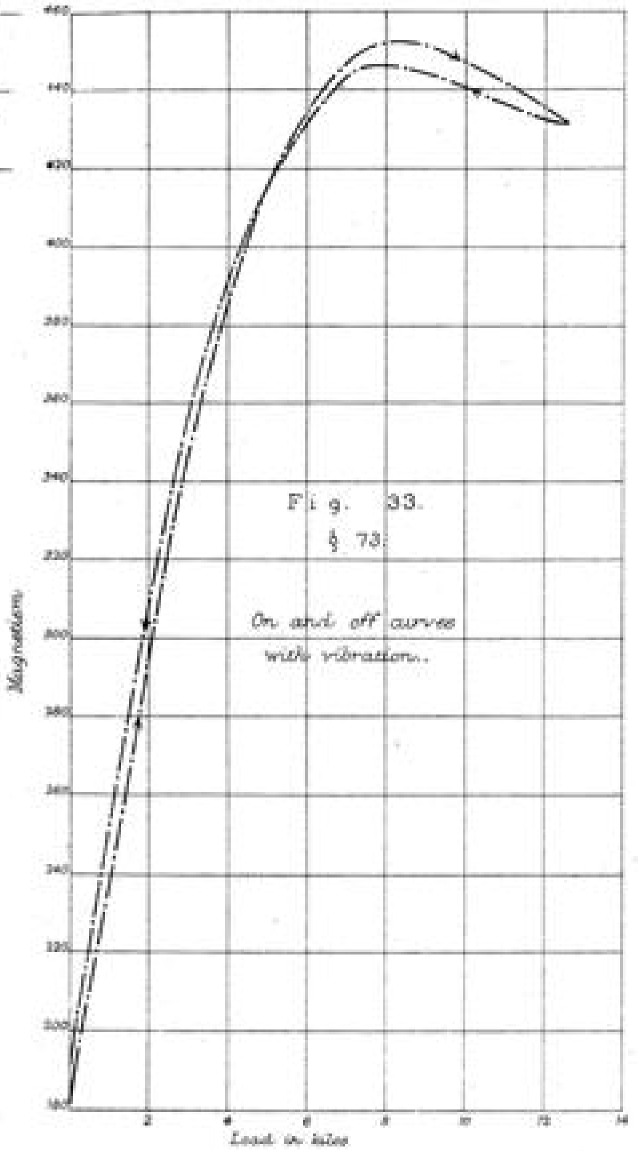


Fig. 33.
§ 73.
On and off curves with vibration.

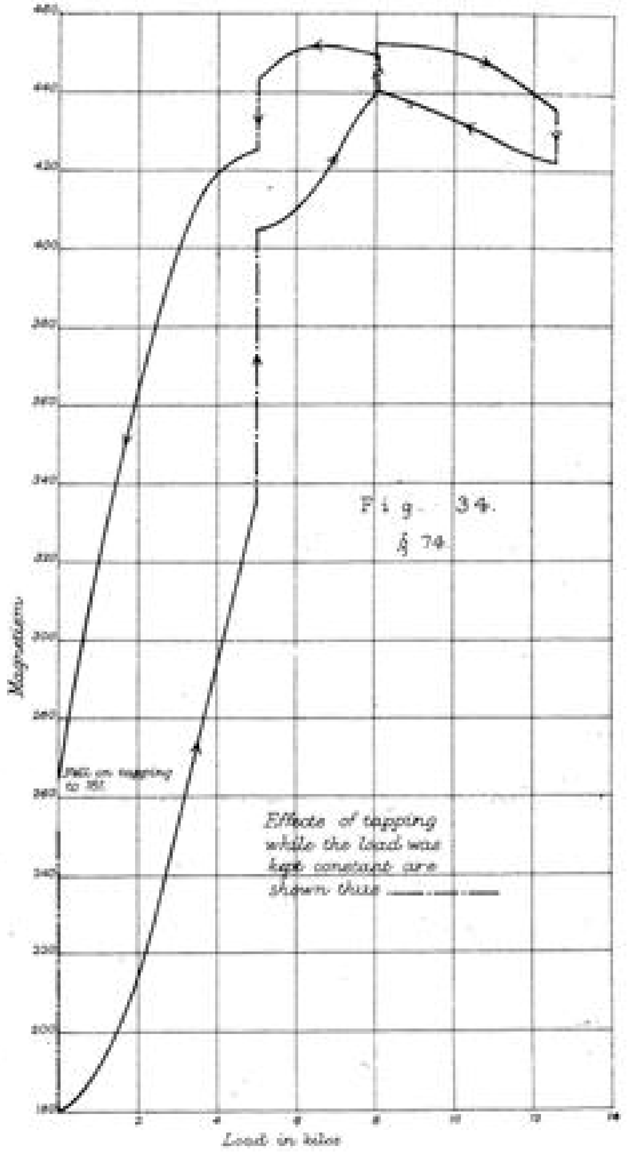


Fig. 34.
§ 74.
Effects of tapping while the load was kept constant are shown thus —·—·—

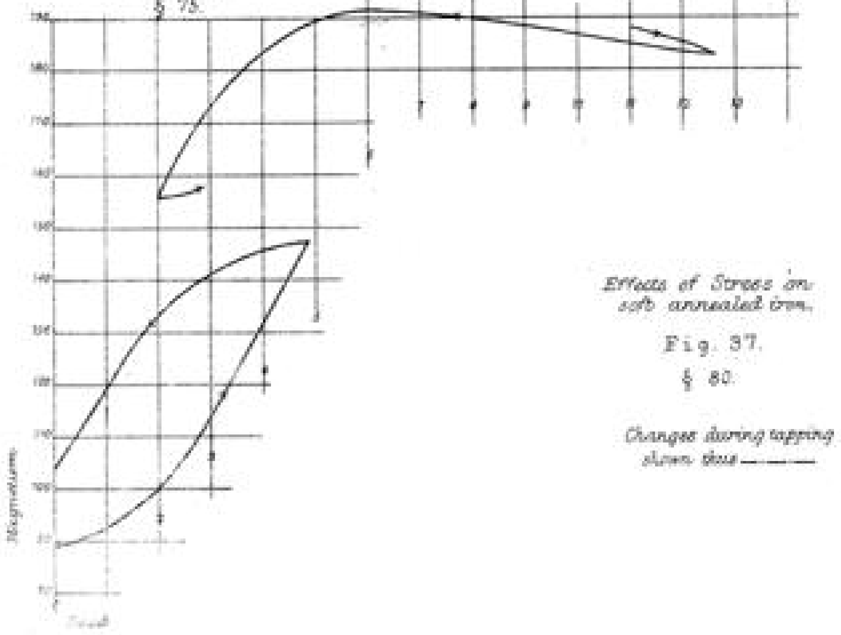


Fig. 35.
§ 75.

Effects of Stress on soft annealed iron.

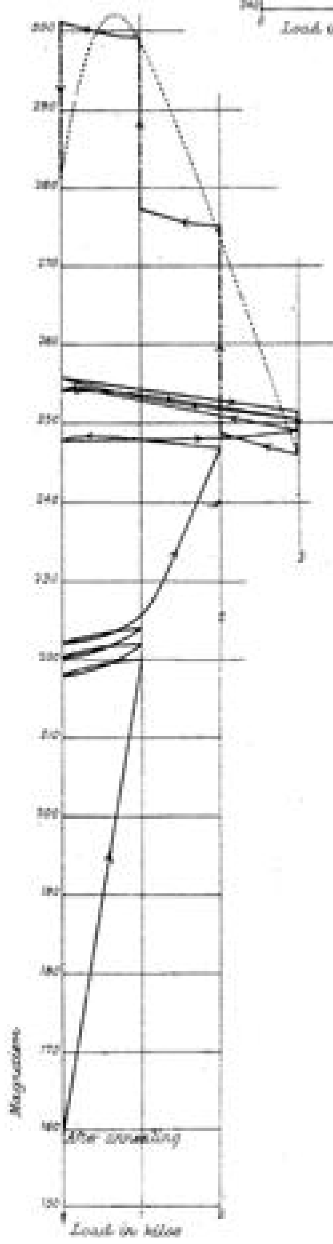


Fig. 37.
§ 80.
Changes during tapping shown thus —·—·—

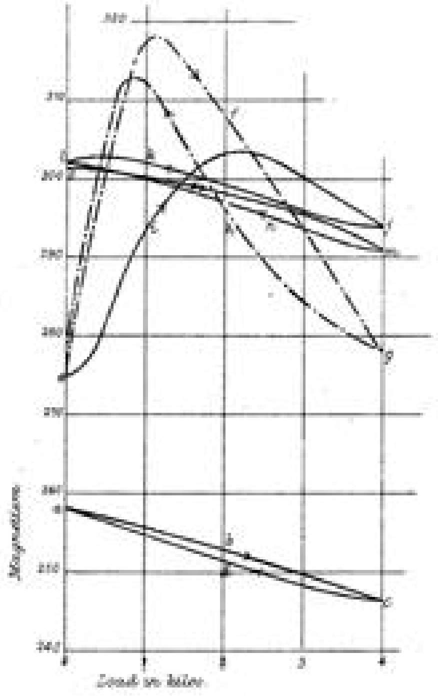


Fig. 38.
§ 82.

Effects of Stress on the Magnetism of Iron in a constant field.

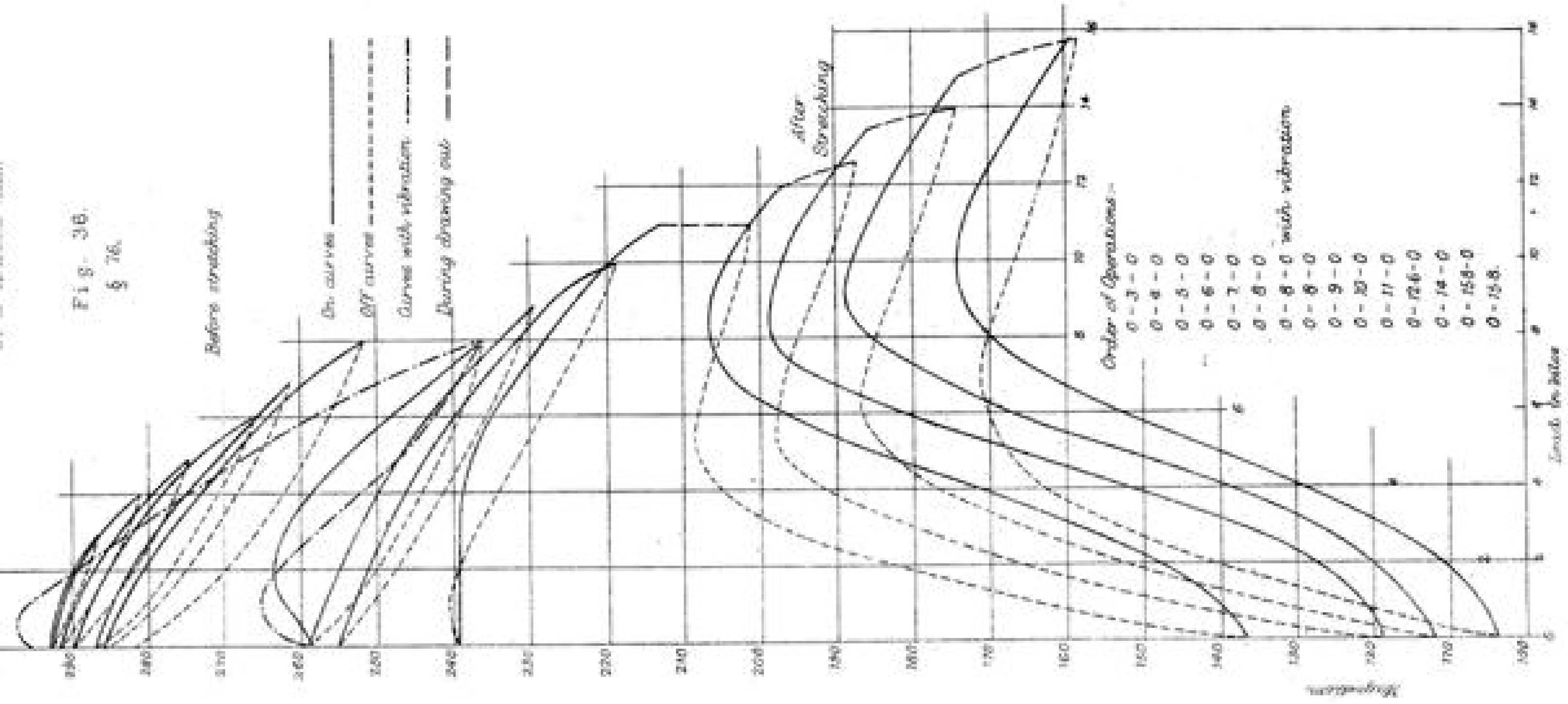


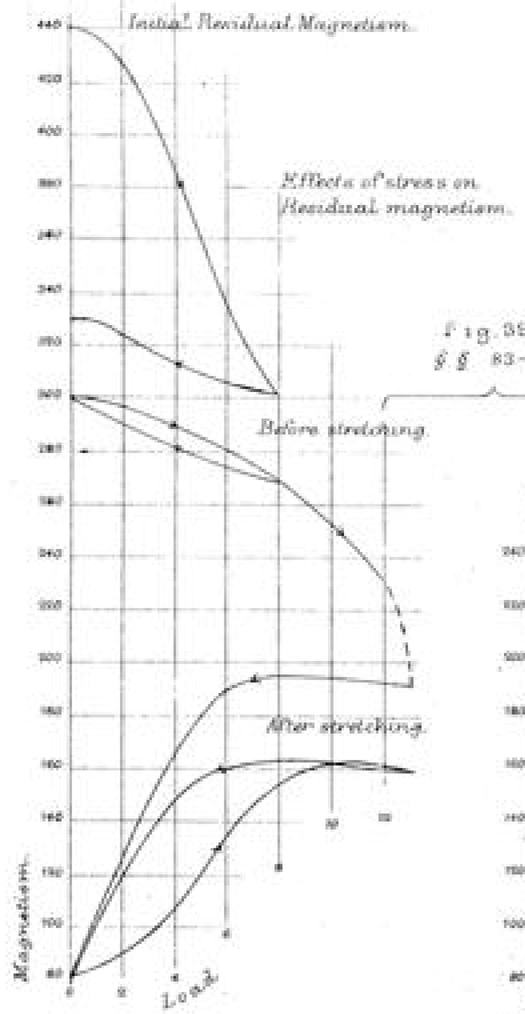
Fig. 36.
§ 76.

West, Newman & Co lith.

Phil. Trans. 1885. Plate 63.

Ewing.

West, Newman & Co. lith.



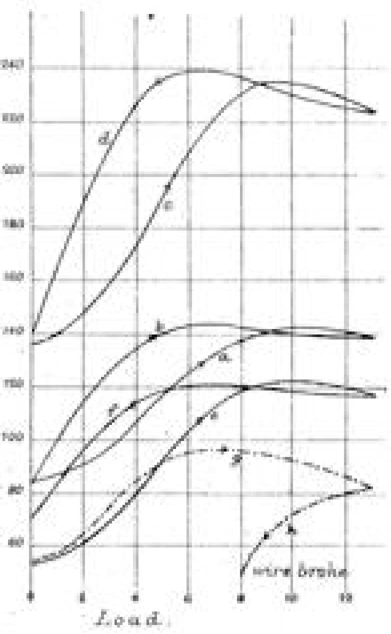


Fig. 39.
§§ 83–85.

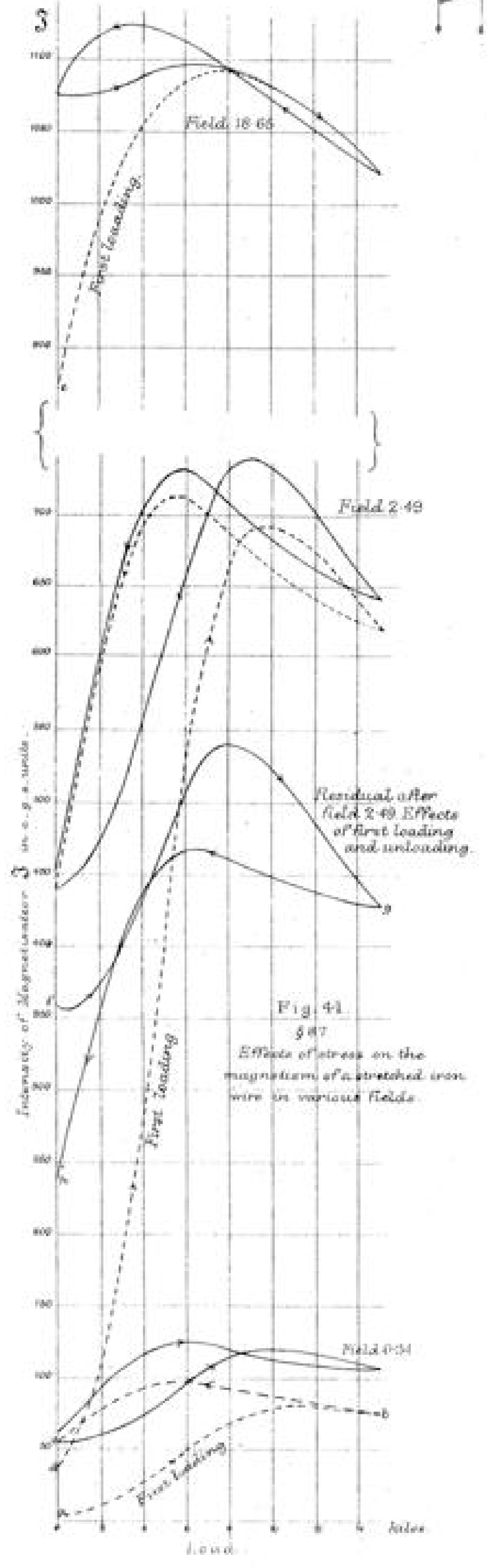
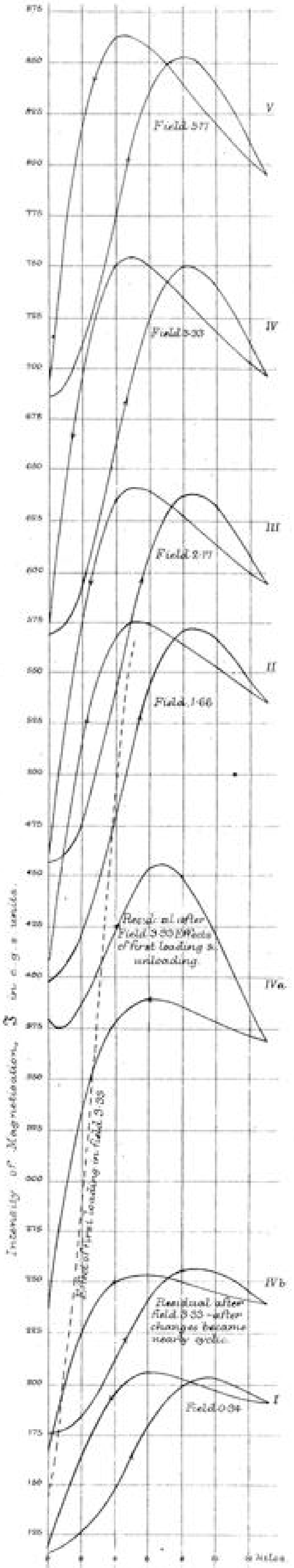


Fig. 41.
§ 87.
Effects of stress on the magnetism of a stretched iron wire in various fields.

Fig. 40.
§ 86.
Effects of stress on the Magnetism of a stretched iron wire in various fields.



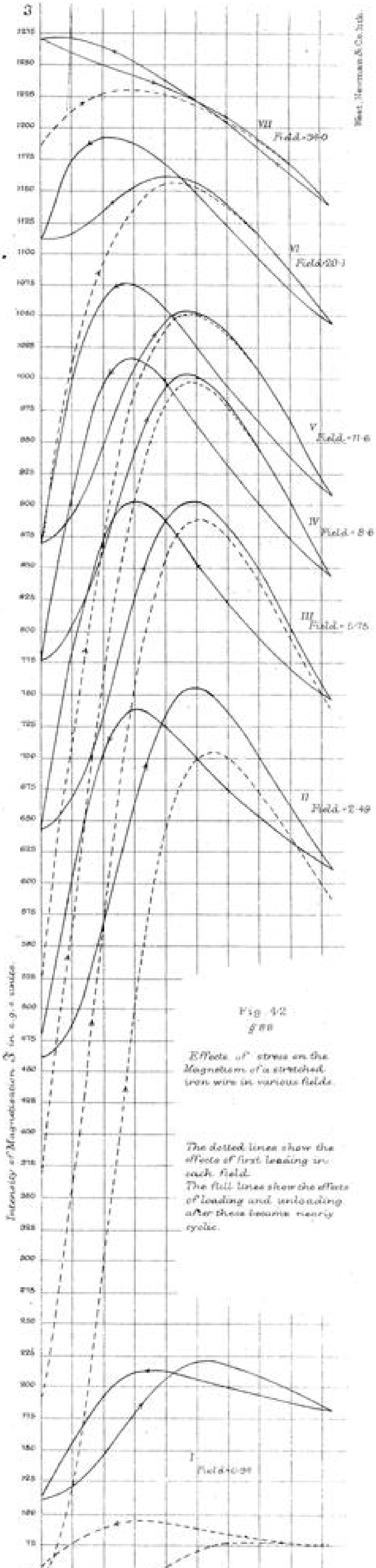


Fig. 42.
§ 88.
Effects of stress on the Magnetism of a stretched iron wire in various fields.

The dotted lines show the effects of first loading in each field.
The full lines show the effects of loading and unloading after these became nearly cyclic.

West, Newman & Co. lith.

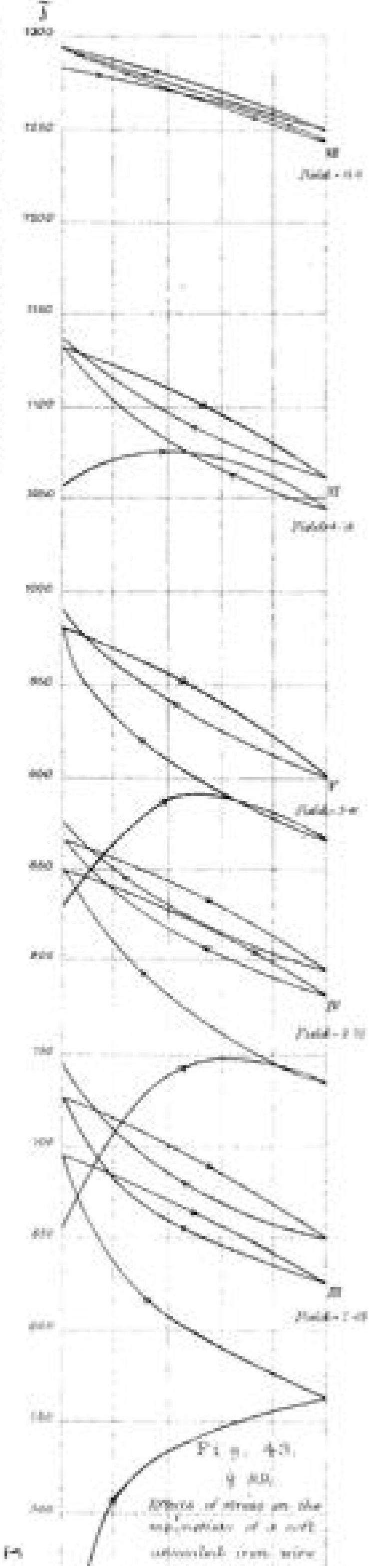


Fig. 43.
§ 89.
Effects of stress on the magnetisation of a soft annealed iron wire in various fields.

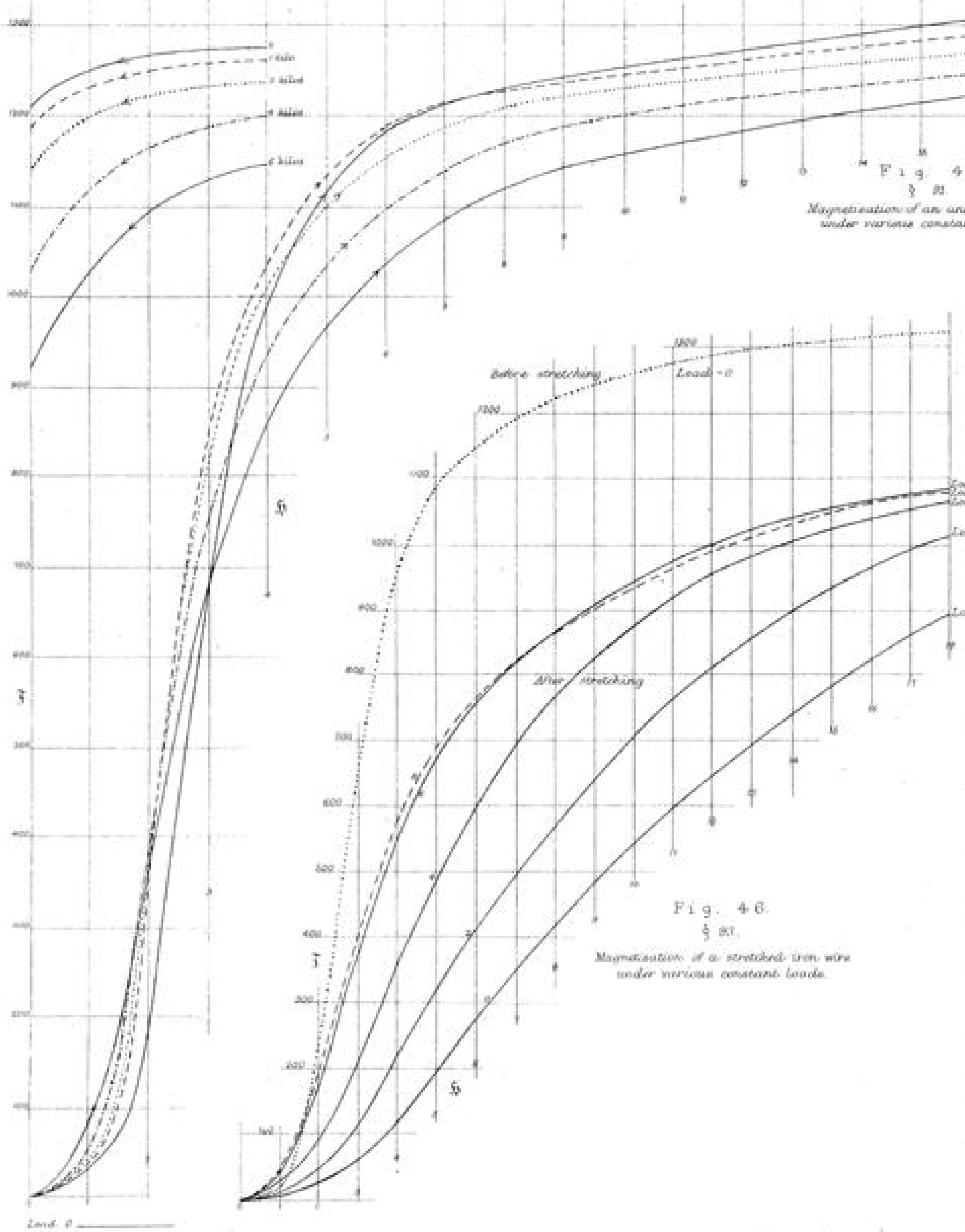


Fig. 44.
§ 91.
Magnetisation of an annealed iron wire under various constant loads.

Fig. 46.
§ 93.
Magnetisation of a stretched iron wire under various constant loads.

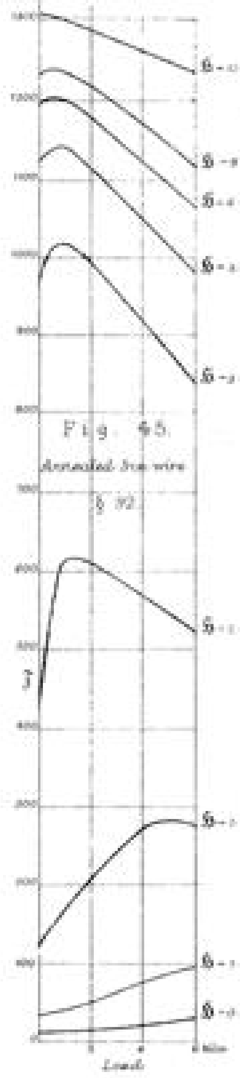
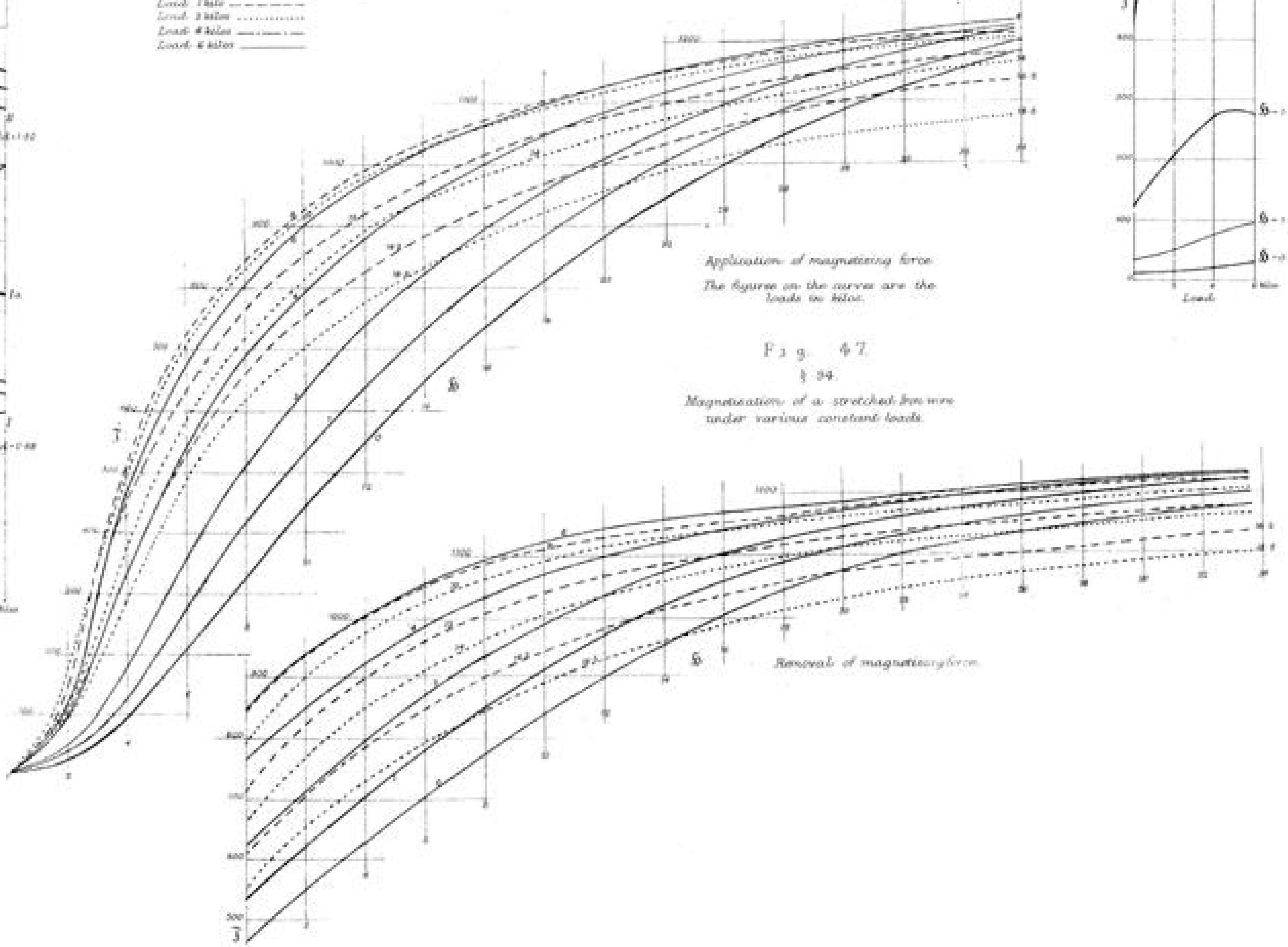


Fig. 45.
Annealed iron wire
§ 92.



Application of magnetising force.
The figures on the curves are the loads in kilos.

Fig. 47.
§ 94.
Magnetisation of a stretched iron wire under various constant loads.

Removal of magnetising force.

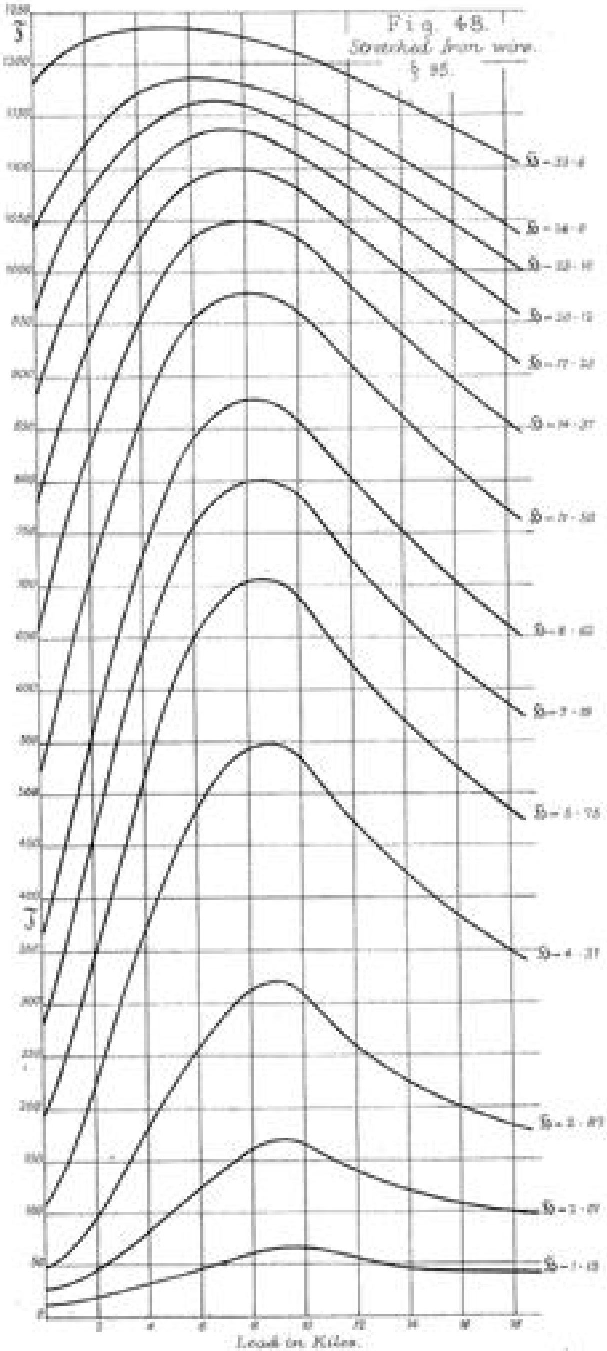


Fig. 48.

Stretched Iron wire.

§ 95.

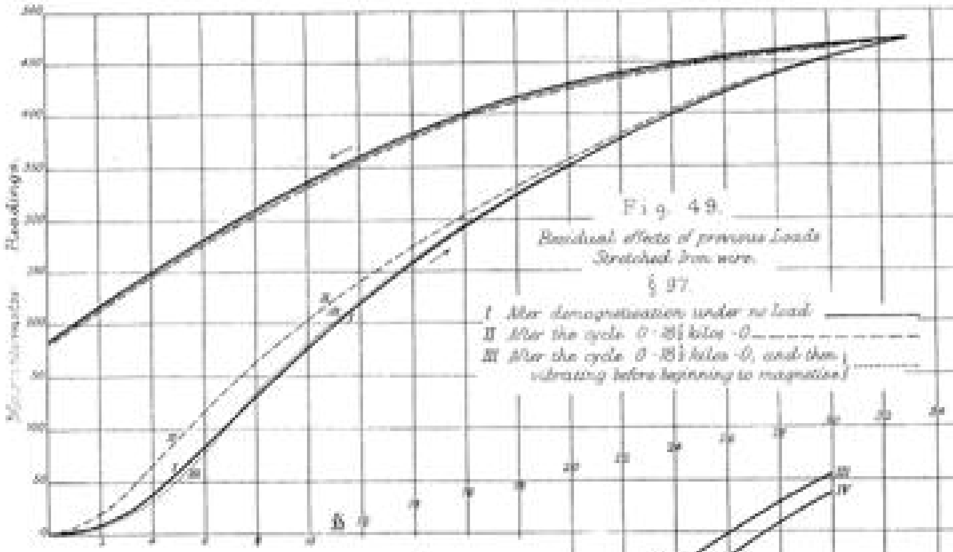


Fig. 49.

Residual effects of previous Loads Stretched Iron wire.

§ 97.

I *After demagnetisation under no load* ————

II *After the cycle 0 - 18½ kilos - 0* – – – – –

III *After the cycle 0 - 18½ kilos - 0, and then vibrating before beginning to magnetise* ……

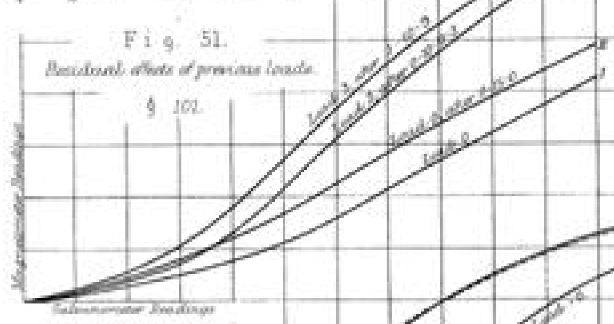


Fig. 51.

Residual effects of previous loads.

§ 101.

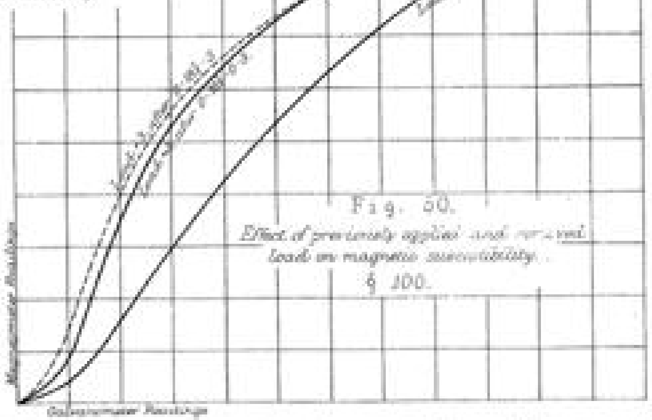


Fig. 50.

Effect of previously applied and removed load on magnetic susceptibility.

§ 100.

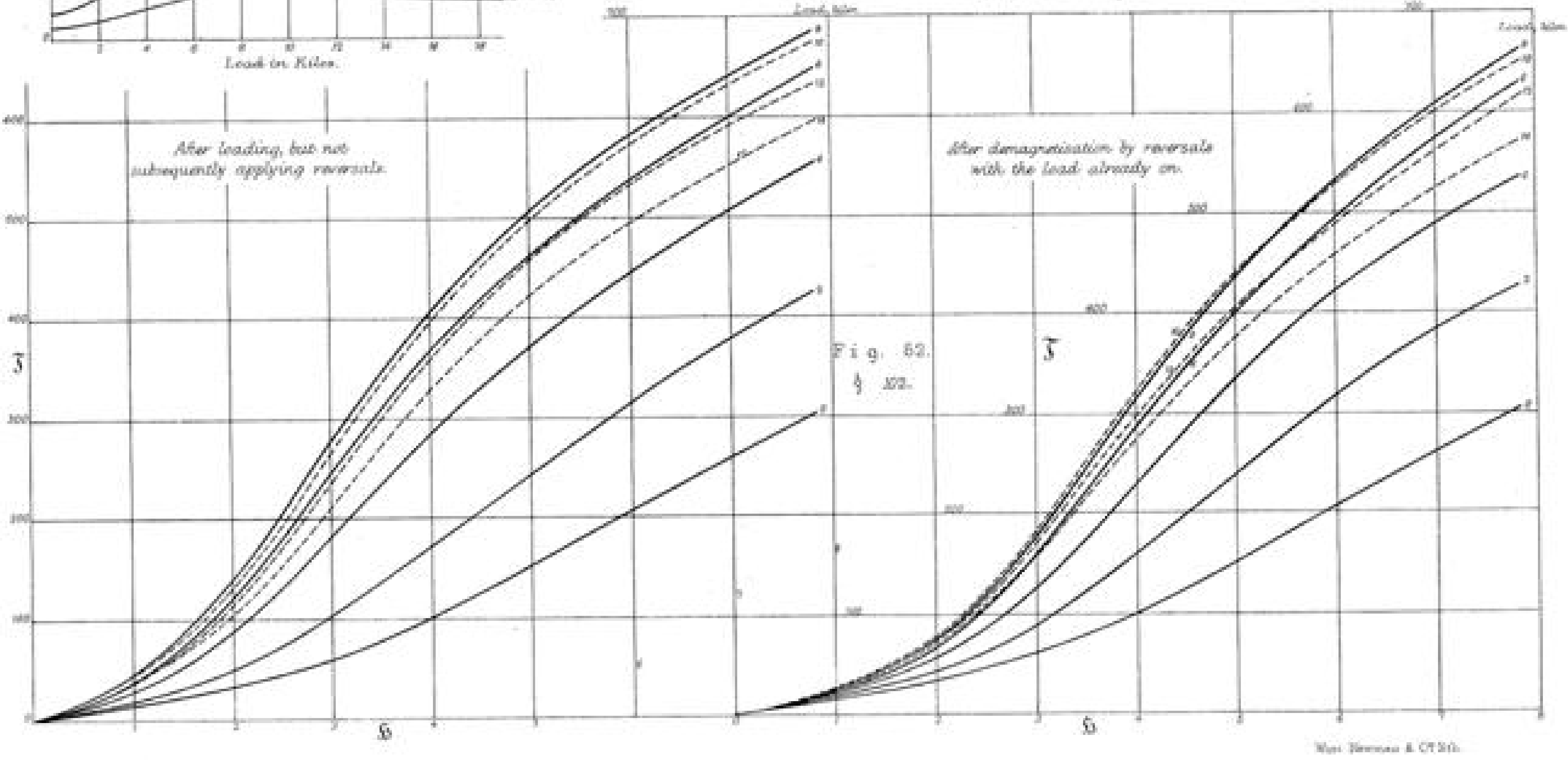


Fig. 52.

§ 102.

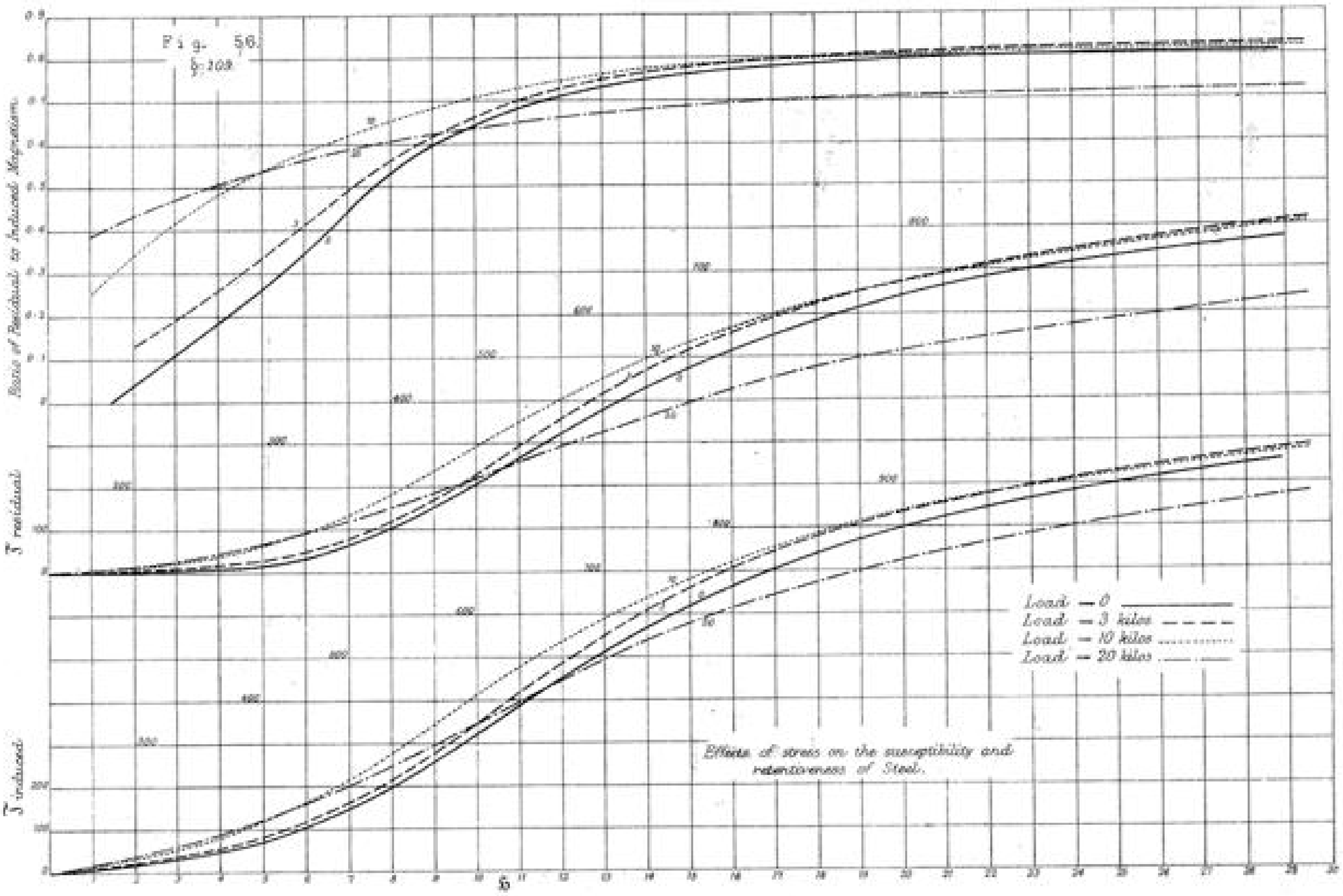


Fig. 56.
§ 109.

Effects of stress on the susceptibility and retentiveness of Steel.

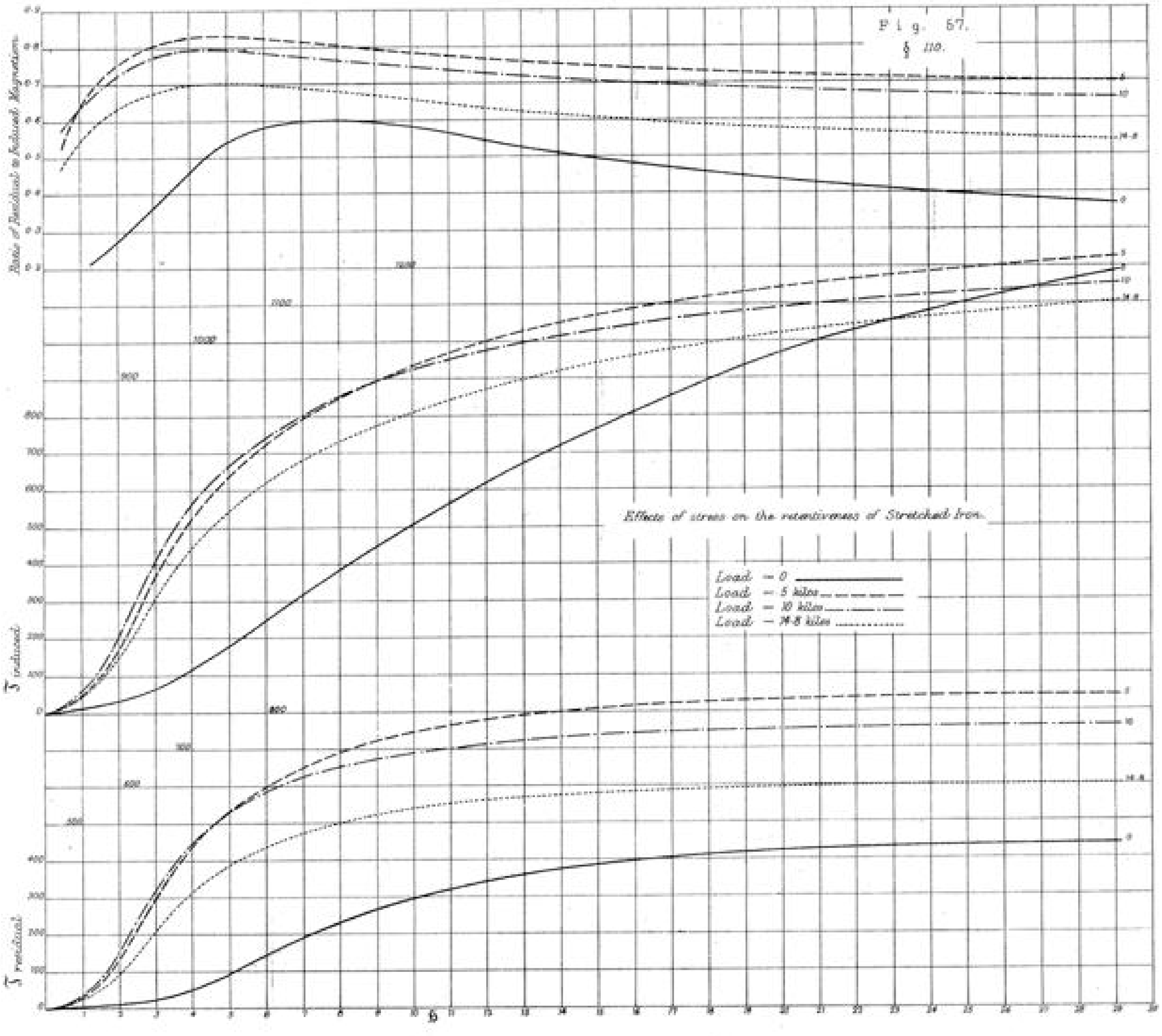


Fig. 57.
§ 110.

Effects of stress on the retentiveness of Stretched Iron.

West, Newman & Co lith.